1. Find a formula for the area of a circular ring whose inner radius is r and whose outer radius is r + 1 (Figure 1). This is a LINEAR FUNCTION because its graph is a straight line. Sketch its graph in an rA-plane

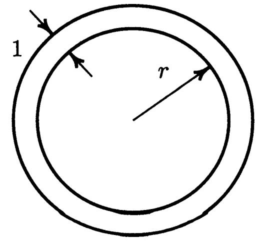


Figure 1

2. **(a)** Give approximate values of $A(1)$ and $A(4)$ for the function $A(x)$ of Figure 2. **(b)** Find the approximate solutions x of $A(x) = 40$ and of $A(x) = 60$ with $0 \le x \le 6$. **(c)** Which is greater $[A(2)]^2$ or $A(2^2)$? Explain.

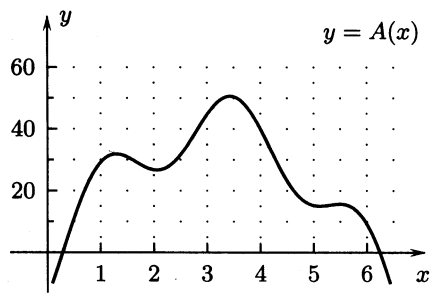


Figure 2

3. The symbols $A(2)$ have one meaning if $A(x)$ is a function of x and another if A represents a constant. Explain.

4. What are $p(1)$, $p(2)$, and $p(3)$ if the points (1, 18), (2 –6), and (3, $\sqrt{2}$) are on the graph of p?

5. Figure 3 shows the graph of the velocity v(t) of a woman's car t seconds after she leaves a parking space. She stops at a traffic light and then goes on the freeway. Give approximate answers to the following questions: **(a)** How long does she wait at the light? **(b)** The speed limit is 55 miles per hour, and her trip is interrupted when she is stopped and given a speeding ticket. When do you imagine she sees the highway patrolman who stopped her? **(c)** During what time interval does she exceed the speed limit on the freeway? **(d)** During part of the short drive between the parking space and the traffic light, she drives at a constant speed. What is it? How far does she travel during that time? (Remember that an hour consists of 60 x 60 = 3600 seconds.)

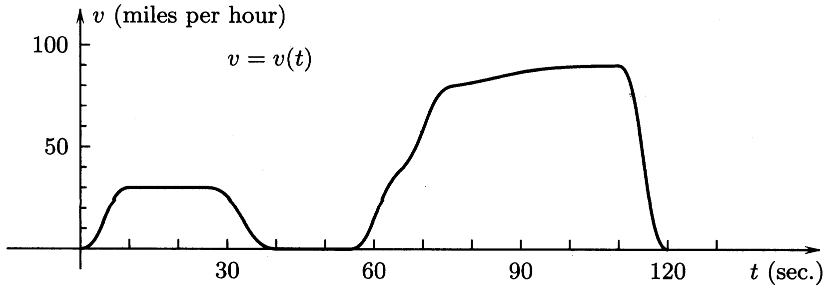


Figure 3

6. The next table gives the number of registered political action committees (PAC's) and the amounts they contributed to candidates for the U. S. Congress during the campaigns for the 1974 through 1988 elections. The contributions have been rounded to the nearest thousand dollars. **(a)** What were the least and greatest increases in the number of PAC's from one election to the next during these years? **(b)** What were the least and greatest increases in the amounts contributed by PAC's? **(c)** What was the average amount contributed per PAC in 1974? In 1988?

POLITICAL ACTION COMMITTEES, 1974 - 1988

Year	1974	1976	1978	1980	1982	1984	1986	1988
Number of PAC's	608	1146	1653	2551	3371	4009	4157	4268
Total contributions (million dollars)	12.527	22.572	34.121	55.217	86.620	105.330	132.671	147.898

7. A function Q has the values $Q(5) = 7$ and $Q(10) = 21$. What are **(a)** $10Q(10)$, **(b)** $Q(6 + 3 + 1)$, **(c)** $Q(100/20)$, **(d)** $Q(10)/Q(5)$, and $Q(Q(5) + 3))$?

8. The average number of cigarettes $C(t)$ smoked per person in the United States is given below for 1945 and every sixth year until 1987. **(a)** What are the change and relative change of $C(t)$ from 1945 to 1963? **(b)** What are the change and relative change of $C(t)$ from 1963 to 1987? **(c)** $C(t)$ has been defined for all t between 1945 and 1987 in Figure 4 by joining the points given in the table with line segments. Find $C(1953)$. This is the average number of cigarettes smoked per capita in 1953 according to the mathematical model in Figure 4. In fact, the average that year was 3500 cigarettes per person. What error is made by using $C(1953)$? What is the relative error (the error divided by the correct value)? **(d)** In Figure 4 the numbers on the vertical axis run from 3000 to 4500. Figure 5 shows the same graph with all of the vertical axis shown from 0 to 4500. Both graphs give good representations of changes in $C(t)$ but the second graph is better for representing relative changes. Explain.

NUMBER OF CIGARETTES SMOKED PER CAPITA, 1945 - 1987

Year	1945	1951	1957	1963	1969	1975	1981	1987
Cigarettes smoked	3500	3700	3700	4300	4000	4200	3900	3300

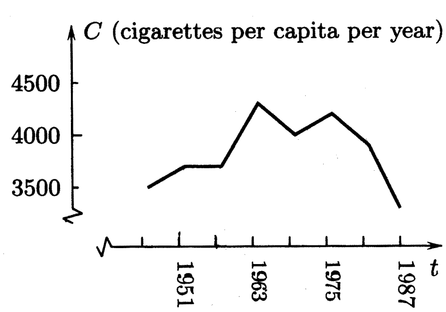


Figure 4

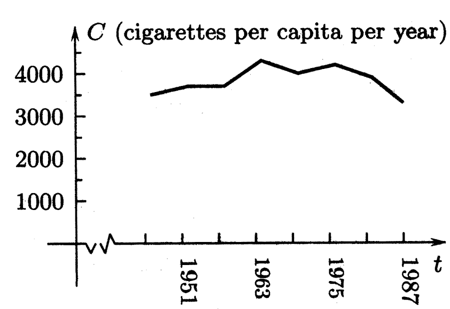


Figure 5

9. Suppose that $P(50) = 2P(1)$ and $P(100) = 3P(50)$. How do you obtain $P(100)$ from $P(1)$? What is the relative change from $P(1)$ to $P(100)$?

10. The next table lists the distances ridden by passengers in private automobiles and in trains and busses in Japan in the years 1960, 1965, 1970, 1975, and 1980. Let $f(t)$ denote the number of kilometers ridden in private cars and $g(t)$ the number in trains and busses in year t. Calculate $A(t) = \dfrac{100 f(t)}{f(t) + g(t)}$ for each of the years in the table. What does $A(t)$ represent?

TRAVEL IN JAPAN, 1960 - 1980 (HUNDRED MILLION KILOMETERS)

Year	Travel by private cars	Travel by trains and busses
1960	1,610	18,581
1965	4,306	26,355
1970	12,221	28,196
1975	17,681	28,319
1980	23,612	27,900

11. Define $F(0)$, $F(1)$, $F(2)$, $F(3)$, $F(4)$, and $F(5)$ so that the relative change of $F(n)$ to $F(n + 1)$ is 3 for $n = 0, 1, 2, 3,$ and 4.

12. One square is circumscribed about a circle of radius 1, and another square is inscribed in the circle (Figure 6). **(a)** What percent of the area of the circumscribed square is not in the circle? **(b)** What percent of the area of the circle is not in the inscribed square? **(c)** How are the areas of the two squares related?

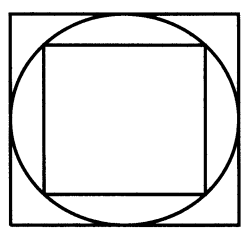


Figure 6

13. Each of Figures 7-10 shows the graph of $y = ax^2 + b$, $y = ax^3 + b$, $y = \dfrac{a}{x} + b$, or $y = \dfrac{a}{x^2} + b$ with integers a and b. Find the equations of the curves.

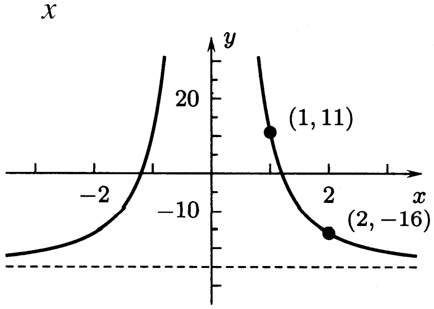


Figure 7

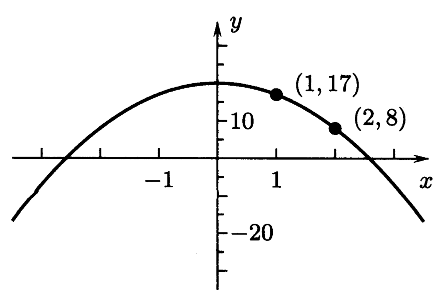


Figure 8

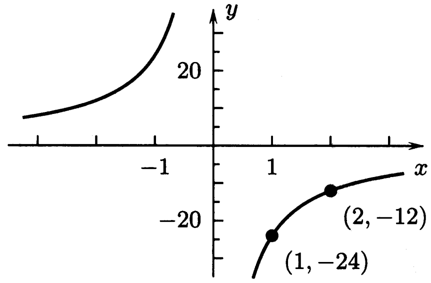


Figure 9

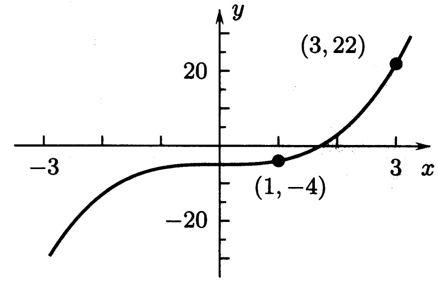


Figure 10

14. Which of x^{-2} and x^{-1} dominates the other **(a)** when x is a large positive or large negative number and **(b)** when x is close to 0?

15. Which are the dominant terms in the numerator and in the denominator of $\dfrac{x^3 - 5}{7x^2 + 3x}$ **(a)** when x is a large positive or large negative number and **(b)** when x is close to 0?

16. **(a)** Which of the lines in Figures 11-13 has the equation $y = ax + b$ with $a > 0$ and $b > 0$? **(b)** Which has the equation $y = ax + b$ with $a < 0$? Is b positive or negative in this case? **(c)** Which has the equation $y = ax + b$ with $b < 0$? Is a positive or negative in this case?

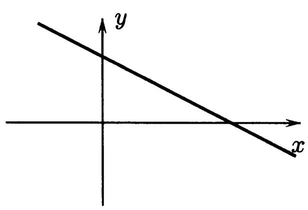


Figure 11

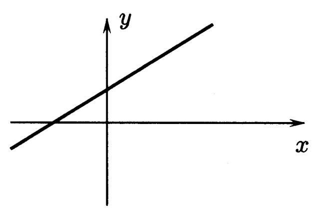


Figure 12

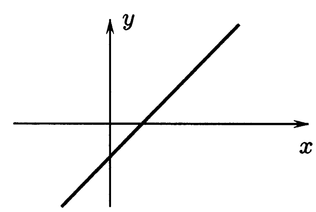


Figure 13

17. Which of the curves in Figures 14-19 is the graph of $y = a + b/x^2$ with $a > 0$? Is b positive or negative? Give your reasoning.

18. Which of the curves in Figures 14-19 is the graph of $y = a + bx^3$ for some constants a and b? Is a positive or negative? Is b positive or negative? Give your reasoning.

19. Which of the curves in Figures 14-19 is the graph of $y = a + b/x$ for some constants a and b? Is a positive or negative. Is b positive or negative? Give your reasoning.

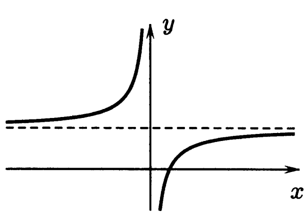


Figure 14

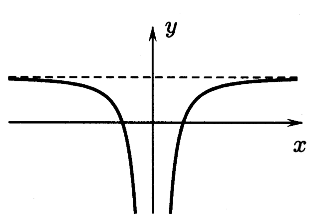


Figure 15

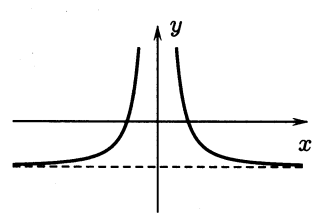


Figure 16

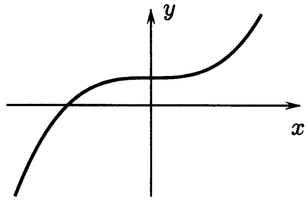


Figure 17

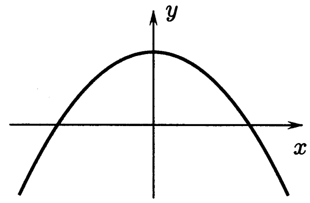


Figure 18

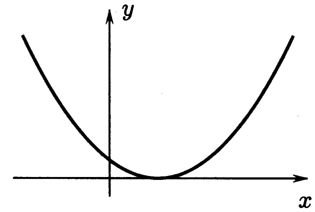


Figure 19

20. A moving van is 200 miles east of El Paso, Texas, at noon and is driving east at the constant velocity of 60 miles per hour. Let s be the distance, in miles, the van is east of El Paso t hours after noon. Write a formula for s in terms of t and graph in an appropriate window.

For questions 21-24, use the following information.

Driving on empty

Imagine that you start on a trip with 20 gallons of gas in your tank, that your car consumes gasoline at the constant rate of 0.05 gallons per mile, and that you drive until your tank is empty without buying any more gas.

21. What mileage does your car get on the trip, measured in miles per gallon?

22. Give a formula for the volume v of gasoline remaining in your tank as a function of the distance s that you have traveled on your trip. What is the domain of this function? Show a sketch of the function over this domain.

23. Suppose you drive at the constant speed of 60 miles per hour. Give a formula for the distance you travel s as a function of the number of hours t that you have traveled. What is its domain? Show a sketch over this domain.

24. **(a)** Use the formulas from Problems 22 and 23 to give a formula for the volume of gas in your tank as a function of t. What is its domain? Show a sketch over this domain. **(b)** What is the rate of change of this function with respect to t? **(c)** How much does the volume of gasoline in the tank change every hour?

25. What is the constant rate of change of $7 - 6x$ with respect to x?

26. Give a formula for the linear function $f(x)$ whose constant rate of change with respect to x is 50 and whose value at $x = 3$ is 10. (Use the point-slope equation for a line.)

27. What is the constant rate of change $470 - \frac{1}{3}z$ with respect to z?

28. Give a formula for the linear function $Z(x)$ such that $Z(0) = 100$ and $Z(5) = 200$.

29. The density of sucrose is 1.58 grams per milliliter. **(a)** Give a formula for the mass w of a sample of sucrose as a function of its volume V. **(b)** What is the rate of change of the mass of a sample of sucrose with respect to its volume? Give the units in which the rate of change is measured.

30. The length L of a copper rod is a linear function of the temperature T. The rod is 100 inches long at 50°F and expands 0.093 inches for every degree increase in its temperature. **(a)** Give a formula for L as a function of T. **(b)** At what temperature is the rod 100.5 inches long?

31. At the surface of the ocean, the water pressure equals the air pressure, which is 14.7 pounds per square inch. In one mathematical model, it is assumed that as you descend vertically into the ocean, the pressure increases at the constant rate of 0.44 pounds per square inch per foot. **(a)** Give a formula for the water pressure p as a function of the depth h beneath the surface with this mathematical model. **(b)** Based on this model, what is the water pressure at the deepest spot in the oceans, Challenger Deep, which is 36,198 feet beneath the surface?

32. A parachutist is 300 meters above the ground at time $t = 0$ seconds and descends at the constant rate of 15 meters per second until she hits the ground. **(a)** Give a formula for her height s above the ground as a function of t and sketch its graph. **(b)** When does she reach the ground? **(c)** The slope of the line in part (a) is the parachutist's upward velocity. What is it?

33. A ball is thrown straight down from a tower with velocity 5 feet per second. We use the mathematical model in which air resistance is disregarded, so that the ball's downward velocity v increases at the constant rate of 32 feet per second per second. **(a)** Give a formula for v as a function of the number of seconds t after the ball was thrown. **(b)** How long does it take for v to increase from 5 feet per second to 37 feet per second? **(c)** When does the ball hit the ground if it hits with velocity 101 feet per second?

34. Figure 20 shows the graph of a linear function $O(V)$ that is a fairly good model of a human's oxygen absorption, measured in liters per minute, as a function of the rate of his or her breathing, also measured in liters per minute. What is the approximate rate of change of oxygen absorption with respect to the rate of breathing?

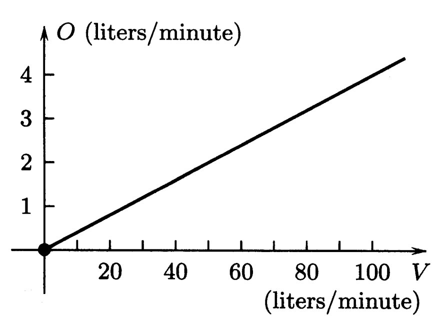


Figure 20

35. A plastic garden hose has an inner diameter of 1 inch and an outer diameter of 1.2 inches. **(a)** A cross section of the hose is the region between two concentric circles. What is its area? **(b)** Give

a formula for the volume of plastic in the hose as a function of its length, measured in inches.
(c) Suppose the plastic costs 5 cents per cubic inch. What is the rate of change of the cost of the plastic in the hose with respect to its length?

36. Which is greater, the rate of change of the circumference of a circle with respect to its diameter or the rate of change of the perimeter of a square with respect to its width?

For questions 37-41, use the following information.

Average velocity

Imagine that a pilot is flying a small plane into a strong headwind toward the west from an airport. As a mathematical model of her flight, we suppose that the plane is $s(t) = t^3 + 30t + 100$ miles from the airport t hours after noon (Figure 21). Note that this function cannot give a perfect description of such a flight, since it would not be possible to know the plane's position with the absolute precision of this formula, and it is unlikely that an actual flight would be even closely described by a formula as simple as this one. We would use this formula—as we use all mathematical models—if it yields results that are as accurate as we need for our particular purposes.

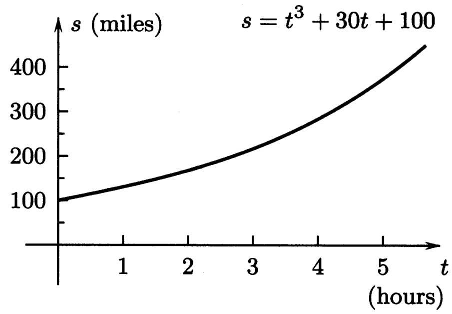


Figure 21

37. Complete the following table of values of $s(t) = t^3 + 30t + 100$.

THE PLANE'S DISTANCE FROM THE AIRPORT

t (hours)	0	1	2	3	4	5
$s(t)$ (KILOMETERS)	100	131	168			

38. What is the plane's average velocity from $t = 1$ to $t = 5$?

39. Calculate the plane's average velocity for $0 < t < 4$ and draw the corresponding secant line of the graph of s(t).

40. Generate $s = t^3 + 30t + 100$ in the four windows, **(a)** $2 < t < 4, 100 < s < 300$, **(b)** $2.5 < t < 3.5, 150 < s < 250$, **(c)** $2.99 < t < 3.01, 216 < s < 218$, and **(d)** $2.999 < t < 3.001, 216.9 < s < 217.1$. Describe the results.

41. Find the plane's average velocity for $2.999 < t < 3$ and for $3 < t < 3.001$. What is the approximate velocity at $t = 3$?

42. An object is at $s = 50 - 50/t$ (yards) on an s-axis at time t (minutes) for $t \geq 1$. Find its average velocity for $1 \leq t \leq 5$. Sketch the graph of $s(t)$ without generating it on a calculator or computer and draw the secant line whose slope is the average velocity.

43. Estimate the velocity at $t = 2$ of the object in Question 42 by calculating its average velocity for $1.99 \leq t \leq 2$, for $2 \leq t \leq 2.01$, and for $1.99 \leq t \leq 2.01$.

44. The graph of a function $W(x)$ is shown in Figure 22. Find the average rates of change of $W(x)$ with respect to x **(a)** for $2 \leq x \leq 5$, **(b)** for $1 \leq x \leq 3$, and **(c)** for $2.50 \leq x \leq 2.51$.

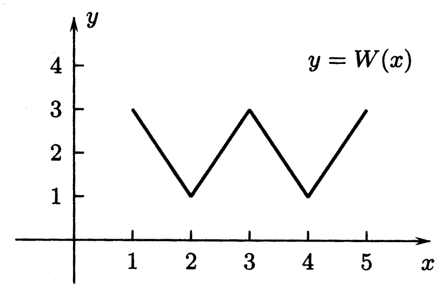


Figure 22

45. A motorcyclist, riding away from her home town, is $s(t) = 10t^2$ (miles) from the city limits at time t (hours) for $0 \leq t \leq 3$. **(a)** What is her average velocity away from the town for $0 \leq t \leq 3$? **(b)** Draw the graph of $s(t)$ and the secant line whose slope is the average velocity from part (a). **(c)** Does she speed up or slow down during the ride? **(d)** What constant velocity would give her the same average velocity for $0 \leq t \leq 3$?

46. A ball rolling toward the edge of a table is $s(t) = 10/t$ centimeters from the edge at time t (seconds) for $t > 1$. **(a)** When is it 5 centimeters from the edge? **(b)** When is it 1 centimeter from the edge? **(c)** What is its average velocity away from the edge for $2 < t < 10$? **(d)** Draw the graph of s(t) and the secant line whose slope is the average velocity from part (c). **(e)** Does the ball fall off the table?

46. A woman's trip is described by the mathematical model in which she is $s(t) = 100 + 50t + 25t^2 - t^4$ miles east of Reno, Nevada on Interstate 80 at time t (hours) for $0 \leq t \leq 5$. **(a)** Generate the graph of s(t) on your calculator or computer in the window $0 \leq t \leq 5$, $-50 \leq s \leq 500$ with s-scale = 50 and use it to describe her trip. **(b)** What is her average velocity toward the east from $t = 0$ to $t = 4$ and from $t = 4$ to $t = 5$? **(c)** Estimate her velocity toward the east at $t = 2$ and $t = 4.5$ by calculating her average velocity toward the east for $2 \leq t \leq 2.001$, and $4.5 \leq t \leq 4.501$.

48. A tank contains $V(t) = 24t - 3t^2$ gallons of water at time t (hours). **(a)** Generate the graph of V(t) in the window $0 \leq t \leq 6$, $-5 \leq v \leq 55$ with V-scale = 10. Use a <maximum> or <trace> command or the graph to find the time t when the volume in the tank is a maximum. (T is an

integer.) **(b)** What is the maximum volume? **(c)** Based on the shape of the graph, is water flowing into the tank more rapidly at $t = 1$ or at $t = 3$? Check your answer by estimating the rate of flow at $t = 1$ or at $t = 3$ with average rates of change of the volume with respect to t over short time intervals including $t = 1$ and $t = 3$.

49. An airplane is 3000 feet above the ground three minutes after taking off from sea level. The air pressure is 14.7 pounds per square inch at sea level and 12.9 pounds per square inch at 3000 feet. What are **(a)** the average rate of change of the plane's altitude with respect to time during the first three minutes of the flight, **(b)** the average rate of change of the air pressure with respect to altitude from sea level to 3000 feet, and **(c)** the average rate of change with respect to time of the air pressure on the plane during the first three minutes of flight? **(d)** How are the answers to parts (a), (b), and (c) related? (Give the units.)

50. Without drawing the graph, find $S(2)$, $\lim_{x\to 2^-} S(x)$, $\lim_{x\to 2^+} S(x)$, and $\lim_{x\to 2} S(x)$ where

$$S(x) = \begin{cases} \sqrt{x+2} & \text{for } x < 2 \\ 14 & \text{for } x = 2 \\ x^8 & \text{for } x > 2. \end{cases}$$

51. Predict $\lim_{x\to 1} \dfrac{x^{1.7}-1}{x-1}$ by finding at least six values of the function for x near 1.

52. What is $\lim_{x\to 8} f(x)$ if $f(8) = 5$ and $f(x)$ is continuous at $x = 8$?

53. Sketch the graph of a function $g(x)$ such that $\lim_{x\to 1} g(x) = 2$ and $g(1) = 3$.

54. Define a function $h(x)$ such that $\lim_{x\to 0} h(x) = \infty$ and $h(0) = 2$. (Use different formulas for $x = 0$ and for $x > 0$.)

55. Without drawing the graph, find $\lim_{x\to 2} g(x)$ and $\lim_{x\to 10} g(x)$ for

$$g(x) = \begin{cases} x^3 + \mathrm{x} & \text{for } x < 2 \\ 10 & \text{for } 2 \le x \le 10 \\ 1 + \sqrt{x+7} & \text{for } x > 10 \end{cases}$$

Justify your answers. What are the most extensive intervals in which $g(x)$ is continuous?

56. Without using a graph, find $\lim_{x\to 10} Z(x)$ and $\lim_{x\to 20} Z(x)$, if they exist, where

$$Z(x) = \begin{cases} x^2 + 900 & \text{for } x < 10 \\ 500 & \text{for } x = 10 \\ x^3 & \text{for } x > 10 \end{cases}$$

57. At what values of x is $f(x)$ discontinuous if

$$f(x) = \begin{cases} x^3 - x^4 + 2 & \text{for} \quad x \le -1 \\ 2x^5 + x + 3 & \text{for} \quad -1 < x < 1 \\ 7 & \text{for} \quad x \ge 1 \end{cases}$$

Justify your answer.

58. What are $\lim_{x \to 2^-} Z(x)$ and $\lim_{x \to 2^+} Z(x)$ if

$$Z(x) = \begin{cases} kx^2 & \text{for} \quad x < 2 \\ x^3 & \text{for} \quad x > 2 \end{cases}$$

For what values of k does $\lim_{x \to 2} Z(x)$ exist?

59. Find constants a and b such that $g(x)$ is continuous for all x, where

$$g(x) = \begin{cases} x^3 & \text{for} \quad x < -1 \\ ax + b & \text{for} \quad -1 < x < 1 \\ x^2 + 2 & \text{for} \quad x \ge 1 \end{cases}$$

60. Find constants a and b such that $h(x)$ is continuous in $[-3, 0]$, where

$$h(x) = \begin{cases} -x & \text{for} \quad -3 \le x \le -2 \\ ax^2 + b & \text{for} \quad -2 < x < 0 \\ 6 & \text{for} \quad x = 0 \end{cases}$$

61. Use at least three difference quotients, to predict the derivative of $\sqrt{x^4 + 3}$ at $x = 1$.

62. What is the approximate value of $T(9.75)$ if $T(10) = 7$ and $T'(10) = 4$?

63. A ball rolling straight down a hill is $s = 1 + \frac{1}{2}t^2$ feet from a tree t seconds after it is released. What are **(a)** its average velocity for $0 \le t \le 2$ and **(b)** its instantaneous velocity at $t = 1$? **(c)** Draw the graph of $s(t)$ and the secant and tangent lines whose slopes are the average and instantaneous velocities from parts (a) and (b).

64. The temperature in a forest is $T = 10 + \dfrac{16}{t^2}$ degrees Fahrenheit t hours after midnight for $1 \le t \le 5$. Which is greater (less negative), the average rate of change of T with respect to t for $1 \le t \le 4$ or the instantaneous rate of change of T with respect to t at $t = 2$? Draw the graph of $T(t)$ and the secant and tangent lines whose slopes are these average and instantaneous velocities.

65. A tank contains $V(t) = 640 - 10t^3$ cubic meters of water at time t (hours) for $0 \le t \le 4$. **(a)** When is water flowing out of the tank at the rate of 270 cubic meters per hour? **(b)** Draw the graph of $V(t)$ with its tangent line at the time from part (a).

66. **(a)** What is the rate of change of the volume $V(w) = w^3$ of a cube with respect to the width w of its sides at $w = 3$? Give the units with w measured in yards. **(b)** Are the average rates of change of the volume with respect to the width for $0 \le w \le 3$ and for $1 \le w \le 3$ less than or greater than the instantaneous rate of change from part **(a)**? **(c)** Illustrate the result of part **(b)** by drawing the curve $V = w^3$ with the relevant tangent line and two secant lines.

67. A walker has gone $s(t)$ (miles) past Reklaw, Texas, t hours after dawn one summer day. What do each of the following values tell about the hike: **(a)** $s(3) = 7$, **(b)** $\dfrac{ds}{dt}(3) = 3$ **(c)** $\dfrac{ds}{dt}(5) = -0.5$ and **(d)** $\dfrac{s(5) - s(0)}{5} = 2$?

68. The median price $N(t)$ (thousand dollars) of new houses and $E(t)$ (thousand dollars) of existing houses on the market in the U.S. in year t AD satisfy **(a)** $N(1991) = N(1970) + 97$, **(b)** $N(1991) = 1.25E(1991)$, **(c)** $E(1991) = 5E(1970)$, **(d)** $\dfrac{dE}{dt}(1991) > 0$, and **(e)** $\dfrac{dN}{dt}(1991) = -0\ 9\ \dfrac{dE}{dt}(1991)$. Rephrase each of these equations as an English sentence without using the symbols $N(t)$ and $T(t)$ for the functions.

69. The amount of tar $T(t)$ (milligrams) and the amount of nicotine $N(t)$ (milligrams) in an average cigarette that was manufactured in the United States in year t AD has the properties **(a)** $T(1957) = 38$, **(b)** $\dfrac{dT}{dt}(1957) = -6$, **(c)** $N(1980) - N(1957) = 24$, and **(d)** $\dfrac{dN}{dt}(1980) = \dfrac{1}{3}\dfrac{dN}{dt}(1957)$. Rephrase each of these equations as an English sentence without using the symbols $T(t)$ and $N(t)$ for the functions.

70. A small airplane whose engine is running at 2500 revolutions per minute has air speed $v(h)$ nautical miles per hour (knots) and uses $g(h)$ gallons of gasoline per hour when it is at an altitude of h thousand feet above the ground. Express the following statements with equations involving $v(h)$, $g(h)$, and their derivatives, under the assumption that the plane's engine is running at 2500 revolutions per minute. **(a)** The Cessna's air speed is 116 nautical miles per hour when it is at an altitude of 4000 feet. **(b)** The Cessna's air speed decreases at the rate of 0.5 knots per thousand feet and its rate of gasoline consumption decreases 0.2 gallons per hour per thousand feet when

the plane is at an altitude of 6000 feet. **(c)** The rate at which the Cessna uses gasoline at an altitude of 12,000 feet is 81% of the rate at 4000 feet.

71. A rock that is propelled straight up into the air at time $t = 0$ (seconds) with no air resistance is $4 + 39.2t - 4.9t^2$ meters above the ground at time t, until it hits the ground. **(a)** From what height is it propelled and with what velocity? **(b)** When is its velocity zero? **(c)** how high does it go?

72. An object is at $s = 100 + 24t - 2t^3$ (feet) on an s-axis at time t. **(a)** When is its velocity zero? **(b)** During what time interval is its velocity positive?

73. A car is $60 + 120t - 80t^{3/2}$ miles east of a town t hours after noon. **(a)** How fast is it traveling and in what direction is it traveling at 12:15? **(b)** When is its velocity zero and how far is it from the town at that time?

74. Use the graph of the function $P(x)$ in Figure 23 **(a)** to estimate $P'(3)$, **(b)** to find the approximate values of x with $0 \le x \le 5$ such that $P'(x) = 0$, and **(c)** to find an approximate value of x such that $P'(x) = 5$.

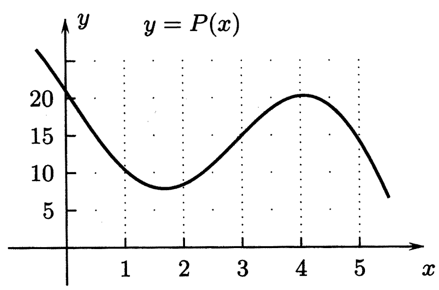


Figure 23

75. Sketch the graph of $J'(x)$ for $J(x)$ of Figure 24.

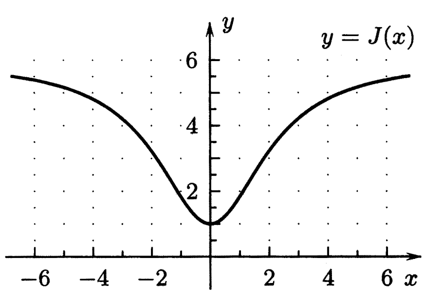
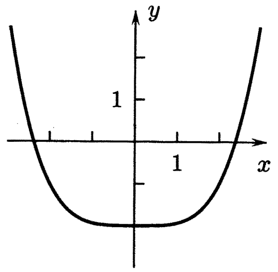


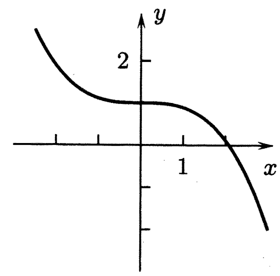
Figure 24

76. Match **(a)** $P(x)$, **(b)** $Q(x)$, and **(c)** $R(x)$ of Figures 25-27 to the graphs of their first derivatives in Figures 25 through 30. Explain your choice.



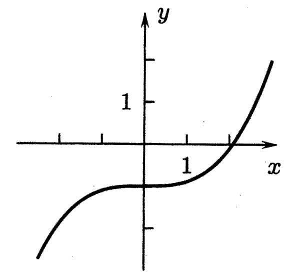
$y = P(x)$

Figure 25



$y = Q(x)$

Figure 26



$y = R(x)$

Figure 27

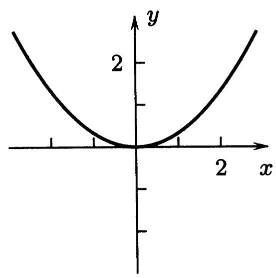


Figure 28

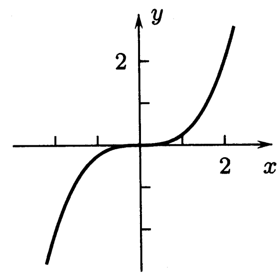


Figure 29

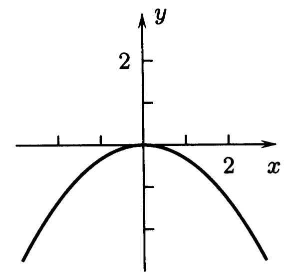


Figure 30

77. Use the values of Q(x) in the next table to estimate $\frac{dQ}{dx}$ (10) and $\frac{dQ}{dx}$ (10.35).

x	9.6	9.8	10.0	10.2	10.4
$Q(x)$	410.9	414.1	417.0	420.2	422.7

78. Figure 31 shows the graph of the temperature $T(h)$ (degrees Celsius) as a function of the altitude h (feet) above sea level one summer day in Los Angeles. This temperature distribution is called an "inversion" because the air is warmer at 3000 feet than at 1000 feet, while normally the air is cooler at higher altitudes. The inversion is caused by warm air from the deserts in the east that blows over the cooler air from the ocean on the west. The temperature inversion traps air pollution near the ground by preventing the upward circulation of warm air. What are the approximate temperature and the approximate rate of change of the temperature with respect to altitude at altitudes of 500, 1500, and 7000 feet?

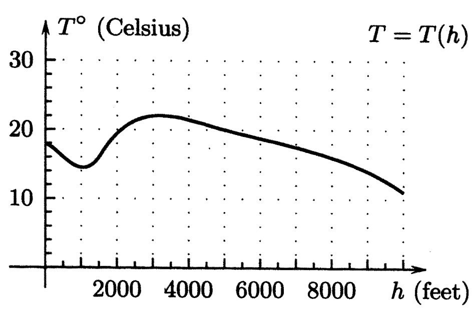


Figure 31

79. Figure 32 shows the graph of a bird's height $h(t)$ above the ground as a function of time. Sketch the graph of its upward velocity dh/dt.

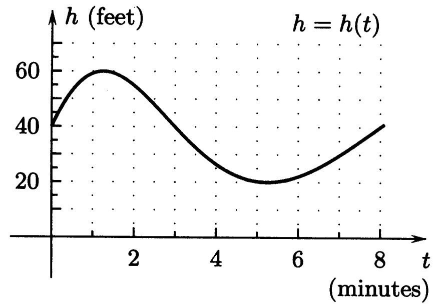


Figure 32

80. Based on the next table of the number of farms $N(t)$ (millions) and total farm acreage $A(t)$ (million acres) in the United States, what were the approximate rates of change with respect to time of **(a)** the number of farms in 1979 and **(b)** the total farm acreage in 1984? **(c)** What was the average size of an American farm in 1976? In 1991?

t (year)	1976	1979	1982	1985	1988	1991
$N(t)$ (millions)	2.497	2.437	2.407	2.293	2.197	2.105
$A(t)$ (million acres)	1054	1042	1028	1012	995	983

81. The percent of water W, measured by weight, in a piece of wood is given as a function of the relative humidity h of the environment by $W(h)$, where $W'(70) = 0.55$. **(a)** Why is it plausible that $W'(70)$ is positive? **(b)** What is the approximate percent of water in the wood at $h = 69$ if $W(70) = 11\%$?

82. The graph in Figure 33 gives the percentage $P(t)$ of the U.S. population whose income was below the government's official poverty level from 1960 to 1990. **(a)** Approximately when was $P(t)$ a minimum for $1960 \le t \le 1990$, and what was dP/dt at that time? **(b)** At what time with $1960 \le t \le 1990$ is dP/dt a minimum (the most negative)? **(c)** What is the approximate minimum value of dP/dt for $1960 \le t \le 1990$?

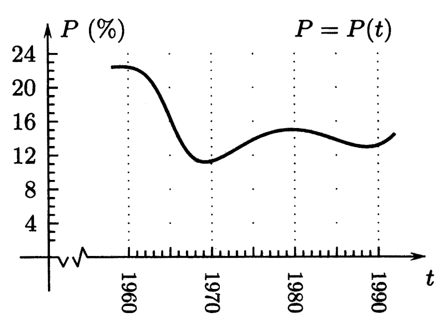


Figure 33

83. Figure 34 shows the upward force (lift) $L(x)$ (pounds) of an airplane wing as a function of the angle x (degrees) it makes with the horizontal. **(a)** What is the approximate rate of change of the lift with respect to the angle at $x = 0$? At $x = 13$? **(b)** What is the approximate maximum lift of the wing and at approximately what angle does it occur? What is the rate of change of the lift with respect to the angle when the lift is a maximum?

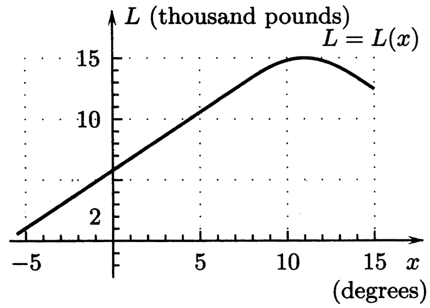


Figure 34

84. Give a formula in terms of the surface area for the rate of change of the volume of a sphere with respect to its surface area.

85. Give a formula in terms of the volume for the rate of change of the surface area of a cube with respect to its volume.

86. A car is $60t + \frac{1}{10}t^5$ (kilometers) past a service station at time t (hours). Where is it and how fast is it traveling when it is accelerating 16 kilometers per hour per hour?

87. A snail is crawling in a straight line toward the north. Figure 35 shows the graph of its velocity v(t) toward the north for $0 \leq t \leq 3$ **(a)** What were its approximate maximum and minimum velocity for $0 \leq t \leq 3$? **(b)** Which of the curves in Figures 36 and 37 is the graph of its acceleration a(t) toward the north? **(c)** The number k on the vertical axis with the correct graph of $a(t)$ is either 1, 5, or 10. Which is it? Justify your answers.

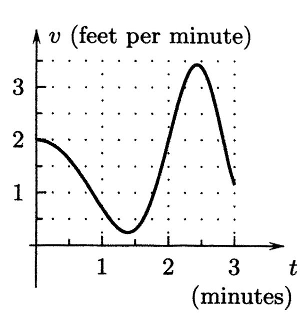


Figure 35

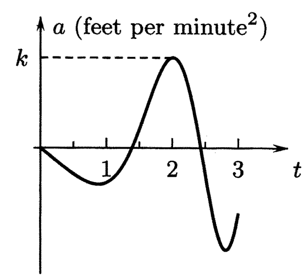


Figure 36

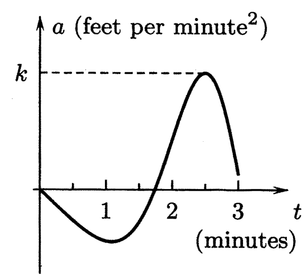


Figure 37

88. According to the February 14, 1994 issue of *Newsweek* magazine, "The rate of increase of health costs nudged downward in the last year, a welcome sign. But costs are still increasing." Rephrase these statements in terms of first and second derivatives.

88. An article in the *Los Angeles Times*, August 26, 1993 had the title "Ozone-depleting emissions drop, study finds." The article, however, stated, "In a paper published in the journal *Nature*, scientists from the National Oceanic and Atmosphere Administration said worldwide emissions of the most widely used chlorofluorocarbons (CFCs) are increasing at substantially reduced rates from previous years." What is wrong with the title of the article?

90. At the beginning of 1990 the total population of the U.S was 248.7 million, of whom 51.3% were women. At that time the total population was increasing at the rate of 3.5 million per year, and the percentage of women was decreasing 0.04% per year. We let $P(t)$ be the total population (measured in millions) in year t and let $F(t)$ be the fraction that were women (the percent divided by 100). Then $P(1990) = 248.7$, $F(1990) = 0.513$, $\frac{dP}{dt}(1990) = 3.5$, and $\frac{dF}{dt}(1990) = -0.0004$.

(a) What was the population of women in the U.S. at the beginning of 1990?
(b) At what rate was the population of women increasing at the beginning of 1990?
(c) At what rate was the population of men increasing at the beginning of 1990?

91. **(a)** Figures 38-39 show graphs of the U. S. national debt $D(t)$ and the U.S. population $P(t)$ as functions of time, Find approximate values of $D(1985)$ and P(1985) from the graphs. Then draw approximate tangent lines and estimate their slopes to find approximate values of $\frac{dD}{dt}(1985)$ and $\frac{dP}{dt}(1985)$. **(b)** Use your estimates from part (a) to give the approximate debt per person and the approximate rate of increase with respect to time of the debt per person at the beginning of 1985.

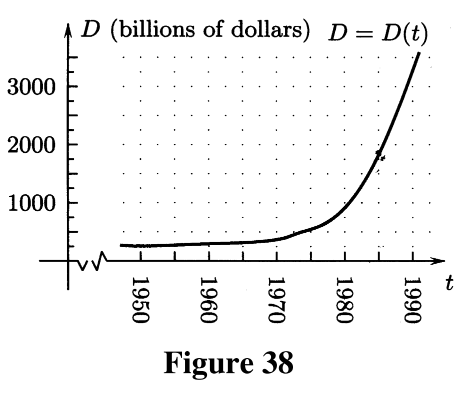


Figure 38

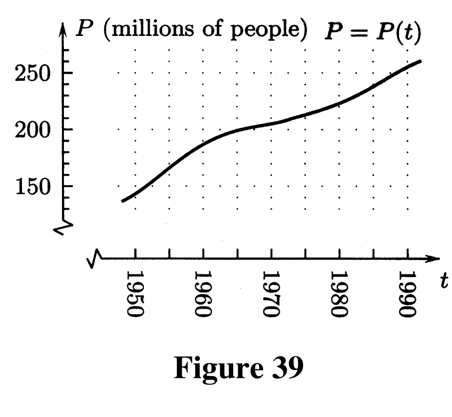


Figure 39

92. **(a)** Express the rate of change dV/dt of the volume $V = \frac{4}{3}\pi r^3$ of a sphere in terms of the radius r and the rate of change dr/dt of the radius. **(b)** At a particular moment the radius of a sphere is 3 inches and is increasing at rate of $\frac{1}{2}$ inch per second. How fast is the volume of the balloon increasing at that moment?

93. $P'(5)$ where $P(x) = R(x)S(x)$, $R(5) = 3$, $S(5) = 4$, $R'(5) = -3$, and $S'(5) = 10$.

94. $\dfrac{dW}{dx}(4)$ where $W(x) = \dfrac{Y(x)}{Z(x)}$, $Y(4) = 2$, $\dfrac{dY}{dx}(4) = 3$, $Z(4) = 5$, and $\dfrac{dZ}{dx}(4) = 6$.

95. $\dfrac{dG}{dx}(3)$ where $G(x) = [y(x)]^{7/2}$, $y(3) = 6$ and $\dfrac{dy}{dx}(3) = -5$

96. Figures 40-41 give the graphs of differentiable function $A(x)$ and $B(x)$. Give approximate values of AB, A/B, and of their first derivatives at $x = 2$.

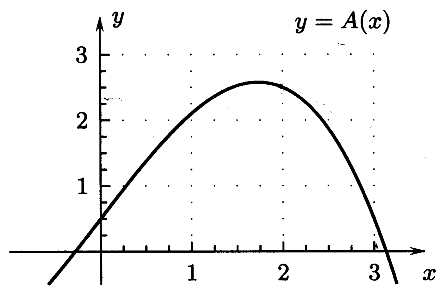


Figure 40

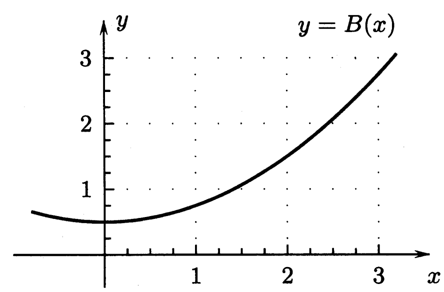


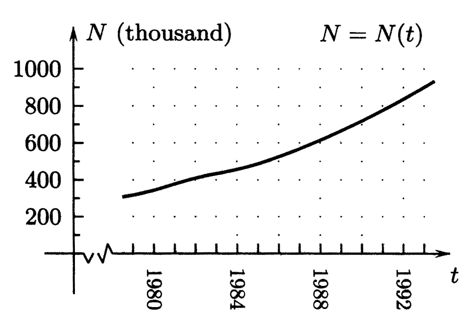
Figure 41

97. $\dfrac{dR}{ds}(1)$ where $R(s) = \dfrac{P(s)}{Q(s)}$, $P(1) = 13$, $Q(1) = -2$, $\dfrac{dP}{ds}$, $\rho(1) = 7$, and $\dfrac{dQ}{ds}(1) = -4$

98. At the beginning of 1991 there were 2.1 million farms in the United States with an average size of 467 acres per farm; the number of farms was Decreasing 0.035 million farms per year; and the average size was increasing 7 acres per farm per year. What was the total acreage of farms and at what rate was it increasing or decreasing at the beginning of 1991?

99. At what rate is the area of a rectangle increasing or decreasing at a moment when it is 5 meters wide and 7 meters long, its width is increasing 3 meters per second, and its length is decreasing 6 meters per second?

100. Figure 42 gives the number of inmates $N(t)$ in U. S. federal prisons and Figure 43 gives the percent $P(t)$ who were incarcerated for drug offences in year t. **(a)** Approximately how many more federal prisoners were held for drug offences in 1993 than in 1979? **(b)** What was the approximate rate of change of the number of federal prisoners being held for drug offences at the beginning of 1988?



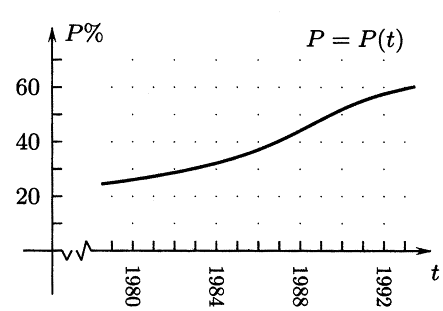


Figure 42

Figure 43

101. At the beginning of 1990, annual health care costs in the U.S. were \$2600 per capita and were rising \$260 per capita per year. At that time the population of the U.S. was 250 million and was increasing at the rate of 2.6 million per year. At what rate were the annual health costs for the entire country increasing at the beginning of 1990?

102. Figures 44-45 give the number $N(t)$ (millions) of MasterCard and Visa accounts and the total outstanding debt $D(t)$ (million dollars) in the U.S. as functions of the year. What were the approximate average debt per credit card and the rate of change of the average debt per credit card at the beginning of 1988?

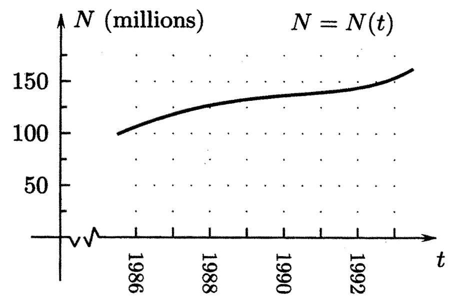


Figure 44

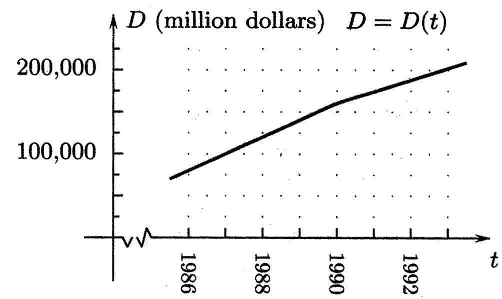


Figure 45

103. Figures 46-47 give the rate $S(t)$ at which shoes were purchased in the U.S. and the U.S. population $P(t)$ as functions of the year. What do $\dfrac{S(t)}{P(t)}$ and $\dfrac{d}{dt}\left[\dfrac{S(t)}{P(t)}\right]$ represent, and what are their approximate values at $t = 1982$?

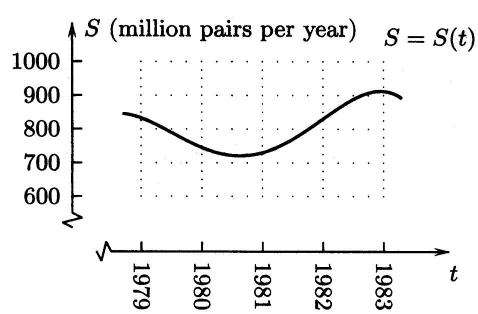


Figure 46

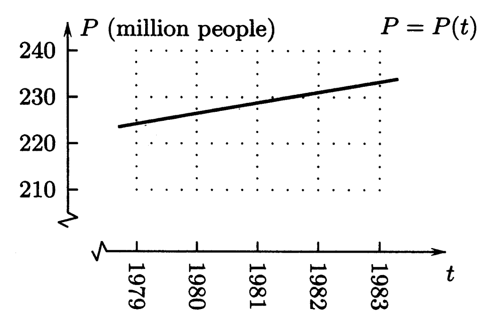


Figure 47

104. What is $P'(3)$ if $P(x) = x^2Q(x)$, $Q(3) = 5$, and $Q'(3) = -6$?

105. What is $\dfrac{dZ}{dy}(9)$ if $Z(y) = \dfrac{R(y)}{S(y)}$, $R(9) = 2$, $\dfrac{dR}{dy}(9) = 4$, $S(9) = 6$, and $\dfrac{dS}{dy}(9) = 8$?

106. What is the derivative of $\frac{G(x)}{x}$ at $x = 2$ if the tangent line to y = $G(x)$ at $x = 2$ has the equation $y = 7 + 2x$?

107. What is the derivative of $W(x)$ at $x = 2$ if $W(2) = 6$ and the derivative of $\frac{W(x)}{x}$ is 4 at $x = 2$?

108. $\frac{dW}{du}(0)$ where $W(u) = \sqrt{Z(u)}$, $Z(0) = 9$, and $\frac{dZ}{du}(0) = 10$

109. $\frac{dB}{dv}(9)$ where $B(v) = [A(v)]^7$, $A(9) = -1$, and $\frac{dA}{dv}(9) = 12$

110. A leaky bucket contains $V(t) = (3 - t)^2$ gallons of water from $t = 0$ (hours) until the bucket is empty. **(a)** When does the bucket have 4 gallons of water in it? **(b)** When is the bucket empty? **(c)** What is the rate of flow out of the bucket at $t = 1$?

111. What is the rate of change with respect to time of the volume $V = w^3$ of an expanding cubic crystal at a moment when its width is 10 millimeters and its width is increasing 3 millimeters per day?

112. The one-dimensional density, measured in mass per centimeter, of a rod of mass 160 grams and length L centimeters is $P = \frac{160}{L}$ grams per centimeter. The rod expands when it is heated. What are **(a)** its density and **(b)** the rate of change with respect to temperature of its density at a moment when it is 40 centimeters long and its length is increasing 0.01 centimeters per degree?

113. At what rate is the radius r of a circle decreasing when the area of the circle is 16 square inches if the area is decreasing 3 square inches per minute?

114. The volume of punch in a hemispherical bowl of radius 10 inches is $10\pi h^2 - \frac{1}{3}\pi h^3$ cubic inches when the punch is h inches deep. **(a)** What are the volume and the rate of change of the volume with respect to h at $h = 5$? **(b)** At what rate is the volume increasing with respect to time at a moment when $h = 5$ inches and h is increasing 6 inches per minute?

115. A powerboat with its engine off has velocity $v = \frac{200}{t + 20}$ feet per second at time t (seconds) for $t \geq 0$. **(a)** When is the velocity 4 feet per second? **(b)** When is the boat decelerating 0.08 feet per second^2?

116. The force of air resistance (drag) on a car is $D = \frac{1}{30}v^2$ pounds when the velocity is v miles per hour. The car is accelerating at a constant rate of 500 miles per hour^2. What is the rate of increase with respect to time of the drag when the car is going 50 miles per hour?

117. Figures 48-49 show the graphs of the width $w(t)$ (meters) and height $h(t)$ (meters) of a rectangle as functions of the time t. What is the approximate rate of change of the area of the rectangle with respect to t at $t = 30$?

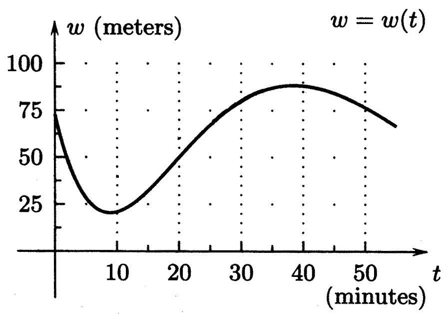


Figure 48

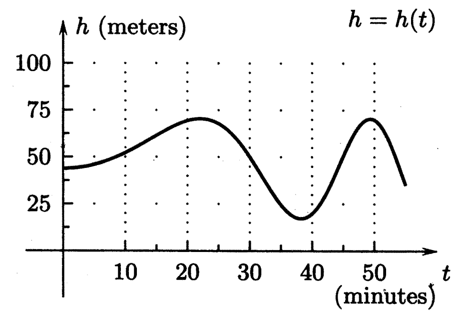


Figure 49

118. Figure 50 shows the graph of the second derivative of a function $g(x)$ such that $g(x)$ and its first and second derivatives are continuous for all x. The second derivative is zero only at $x = -1$ and $x = 4$. **(a)** What are the most extensive open intervals in which the graph of $g(x)$ is concave up and in which it is concave down? **(b)** Does the graph of $g(x)$ have any inflection points? If so, at what values of x?

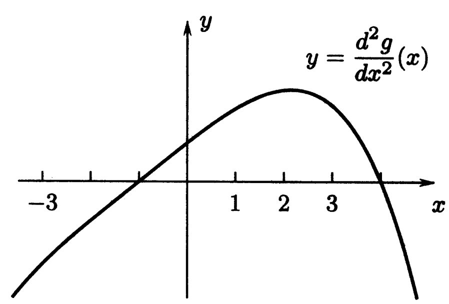


Figure 50

119. Figure 51 shows the graph of the second derivative of a function $L(x)$ that is continuous and has continuous first and second derivatives in [0, 12]. **(a)** What are the most extensive open subintervals of [0, 12] in which the graph of $L(x)$ is concave up and in which it is concave down? **(b)** At what values of x with $0 < x < 12$ does the graph of $L(x)$ have inflection points?

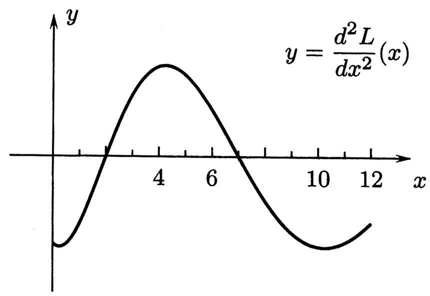


Figure 51

120. The function $g(x)$ and its derivatives are continuous for all x, and $g''(x) = x(x-1)^2(x-2)^3$. What are the most extensive open intervals in which the graph of $g(x)$ is concave up and in which it is concave down? At what values of x does it have inflection points? Justify your answers.

121. $K(x)$ is continuous for all x, and $K'(x) = 4x - x^2$. What are the most extensive intervals in which $K(x)$ is increasing and decreasing? Where is its graph concave up and concave down? Where does $K(x)$ have local maxima or minima and where does its graph have inflection points? Justify your answers.

122. Figure 52 shows the graph of the rate of change dV/dt of the volume of water in a tank for $0 \le t \le 8$. **(a)** What is the maximum rate of flow into the tank? Out of the tank? **(b)** When is the volume of water in the tank increasing? Decreasing? **(c)** When is the rate of flow of water into the tank increasing? Decreasing? **(d)** At what time with $0 < t < 8$ does $V(t)$ have a local maximum? A local minimum?

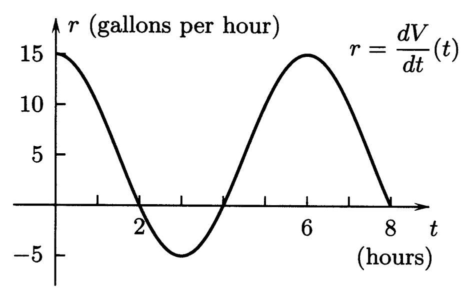


Figure 52

123. For what values of the parameter a does $x^3 - 3ax$ have a local maximum and a local minimum? Give formulas for the local maximum and minimum in terms of a.

124. Suppose that $G'(x) \ge 0$ and $G''(x) < 0$ for $0 \le x \le 10$. Which is the greatest and which is the least of $G'(0)$, $G'(10)$, and $\dfrac{G(10) - G(0)}{10}$?

For questions 125-127, use the following information:

Grow all you can: finding a maximum area

Imagine you want to enclose a rectangular garden using a dormitory wall as one side and up to 40 feet of fence for the three other sides (Figure 53) and that you want to make the area of the garden as large as possible.

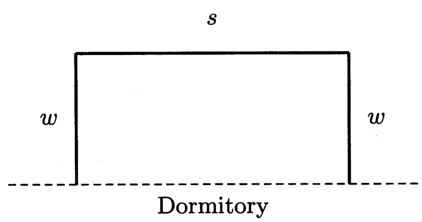


Figure 53

125. **(a)** Explain why you would have to use all 40 feet of fence to maximize the area of the garden and why in this case $s = 40 - 2w$. **(b)** Calculate s and the area of the garden for the choices of w in the table below, under the condition that all 40 feet of fence is used. Describe how s and the area change a w runs through the values in the table. **(c)** What is wrong with using $w = -10$ or $w = 30$?

w	0	4	8	12	16	20
$s = 40 - 2w$	40	32				
Area	0	128				

126. **(a)** Give a formula for the area $A(w)$ of the garden as a function of w, under the assumption that all 40 feet of fence is used. What is the domain of this function? (Allow for zero areas.)
(b) Generate the graph of $A(w)$ in the window $-5 \le w \le 25, -50 \le A \le 250$ with w-scale = 10 and A-scale = 50, and use a <maximum> or <trace> operation on your calculator or computer to predict the greatest possible area of the garden and the corresponding dimensions.

127. What is the length s of the garden that yields the greatest possible area? What is the greatest possible area?

For questions 128-130, use the following information.

Minimizing a cost

Imagine that we need to make a rectangular box with a square base that has a volume of 20 cubic meters (Figure 54) and that we are to use materials that costs 4 dollars per square meter for the base and top and 2 dollars per square meter for the four sides. What dimensions should we give the box to minimize the cost of material? To answer this question, we let w denote the width o the square base and h the height of the box, measured in meters. Then the volume of the box is w^2h, its base and top each has area w^2, and each of the four sides has area wh.

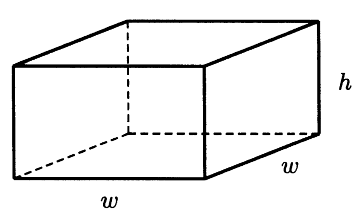


Figure 54

128. **(a)** What is the height h of the box if its volume is 20 cubic meters and its base is 1/2 meter wide? What is the cost C of material for the base, top, and four sides in this case? **(b)** What is the height h and cost C if the volume is 20 cubic meters and the base is 4 meters wide? **(c)** What is the height h and cost C if the volume is 20 cubic meters and the base is w meters wide? This gives formulas for h and C as functions of w. Generate the graph of $C(w)$ for $-0.5 < w < 5.5$, $-35 < C < 350$ with A-scale = 100. You should obtain the curve in Figure 55.

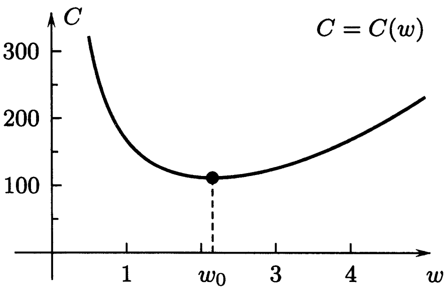


Figure 55

129. **(a)** What happens to $C(w)$ as $w \to \infty$ and as $w \to 0^+$, and what does this say about the cost of possible boxes? **(b)** Use a minimum or trace operation on your calculator or computer to find the approximate minimum of $C(w)$ and the approximate value of w that gives that area.

130. What is the height of the box that costs the least and how much does that box cost? Check your answer by comparing it with the results of PROBLEM 129.

131. **(a)** Give a formula for the perimeter of a rectangle as a function of its width under the condition that its area is 7 square feet. What is the domain of this function? **(b)** Give a formula for the area of a rectangle as a function of its width under the condition that its perimeter is 100 inches. What is the domain of this function?

132. A rectangular box has a square base and a square top. **(a)** Give a formula for the volume of the box as a function of the width of its base under the condition that the total area of its six sides is 24 square feet. What is the domain of this function? **(b)** Give a formula for the total area of its six sides as a function of the width of its base under the condition that its volume is 9 cubic meters. What is the domain of this function?

133. **(a)** What are the volume V, the area A_B of the base, and the area A_L of the lateral (curved) sides of a right circular cylinder of radius r and height h? (Figure 56) **(b)** What is the total area A of its base, top and curved sides?

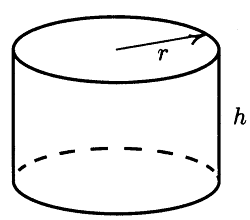


Figure 56

134. A rectangular box with a square bottom and top is to have a volume of 1000 cubic inches. The four sides are to be made from material that costs two cents per square inch at the top and bottom are to be made from material that costs three cents per square inch. Give a formula for the cost of the box as a function of its width.

135. In order for a package to be mailed by parcel post, its length (the length of its longest side) plus its girth (the perimeter of the cross section perpendicular to its longest side) must be no greater than 108 inches (Figure 57). **(a)** Why does the rectangular box of maximum volume that can be shipped by parcel post have its length plus its girth equal to 108 inches? Why does it have square cross sections? Suppose that a box meets these conditions. **(b)** What is the length of the longest side if the square cross sections are 10 inches wide? What is the volume of the box in this case? **(c)** What is the length of the longest side and the volume V if the square cross sections are w inches wide? **(d)** Find the dimensions of the box of maximum volume. Justify your answer.

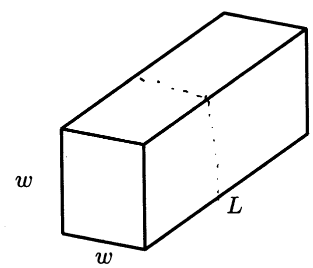


Figure 57

136. A farmer uses 80 feet of fence to construct three rectangular pens as in Figure 58. **(a)** What is the depth y of the pens if all of the fence is used and the overall width x across the front of the three pens is 10 feet? What is the total area of the pens in this case? **(b)** What is the depth y of the pens if all the fence is used and $x = 30$ feet? What is the total area of the pens in this case? **(c)** What is the depth y of the pens and their total area $A(x)$ if all of the fence is used and the overall width of the pens is x feet? **(d)** Find the dimensions that give the maximum total area. Justify your answer. As a partial check, generate the graph of $A(x)$ in the window $-5 < x < 45, -25 < A < 250$ with x-scale = 10 and A-scale = 50.

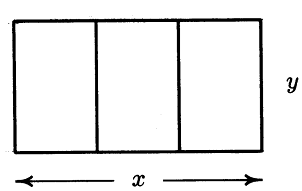


Figure 58

137. A cylindrical can with no top is to be constructed so its volume is 8π cubic feet. **(a)** Give a formula for the total area $A(r)$ of its base and curved sides as a function of the radius of its base. **(b)** Find the dimensions that minimize $A(r)$. Justify your answer. Generate the graph of A(r) in the window $-3 < r < 5, -10 < A < 100$ with A-scale = 10 as a partial check of your answer.

138. A rectangle is to have its base on the x-axis and its upper corners on the parabola $y = 36 - x^2$ as in Figure 59. **(a)** What are the width, height and area of the rectangle if its right side is at $x = 4$? **(b)** For what values of $x > 0$ is the area zero? **(c)** What are the width, height and area of the rectangle if its right side is at x with $0 < x < 6$? **(d)** Find the dimensions that maximize the area of the rectangle. **(e)** Of all such rectangles, what are the width and height of those with minimum area? Justify your answer.

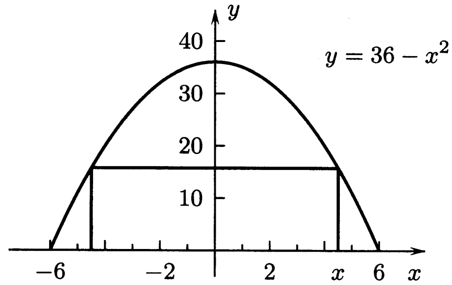


Figure 59

139. A triangular prism with vertical sides and with horizontal top and bottom that are right triangles with sides of lengths $3x$, $4x$, and $5x$, is to be constructed so its volume is 12 cubic feet (Figure 60). **(a)** Give formulas for its height h and the total area A of its top, bottom, and three sides as functions of x. Justify your answer. **(b)** For what value of x is A a minimum?

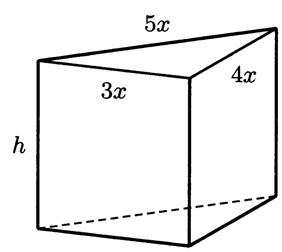


Figure 60

140. A rectangular garden is to have an area of 150 square feet. The fencing for three sides costs \$5 per foot and the fencing for the fourth side costs \$10 dollars per foot. **(a)** Give a formula for the cost of the fence as a function of the length of the side that costs \$10 per foot. **(b)** What is the minimum cost of the fence?

141. A rectangular box is to have volume $\frac{8}{3}$ cubic feet and be twice as long as it is wide. What dimensions should it have to minimize the total area of its six sides?

142. A woman wants to build a rectangular garden with an ornamental fence that costs \$2 per foot for three sides and \$3 per foot for the fourth side. She will spend \$120 for the fence. What should the dimensions of the garden be to maximize its area?

143. A rectangular box with no top is to be 5 feet wide and have a volume of 6 cubic feet. **(a)** Give a formula for the total area A of its bottom and four sides as a function of its length. **(b)** How long and how tall should it be to minimize the total area of its four sides and bottom?

144. A rectangular box with a square bottom and no top is to have a volume of six cubic feet. The material for the bottom costs three dollars per square foot and the material for the four sides costs two dollars per square foot. How wide and how tall should it be to minimize its cost?

145. A cylindrical can with no top is to be made from 12π square inches of tin. What should be the height of the can and the radius of its base to maximize its volume?

146. A cylindrical can with a top is to have a volume of 2π cubic meters. What should be the dimensions of the can to minimize the total area of its top, bottom, and curved sides?

147. A printer is to use a page that has an overall area of 80 square inches with margins of one inch at the bottom and sides and 1 1/2 inches at the top. What should be the dimensions of the paper so that the area of the printed portion is a maximum?

148. A boy has enough money to buy 600 square inches of material to make a box with square ends and no top. He wants to paint the ends red, but only has enough paint if they are no larger than 11 inches on each side. What dimensions should he give the box for it to have the maximum volume?

149. A horse breeder wants to use 180 feet of fencing for a rectangular corral that extends at least across the entire front of a 100-foot-long stable, as in Figure 61. What dimensions should he give the corral for it to have the maximum area?

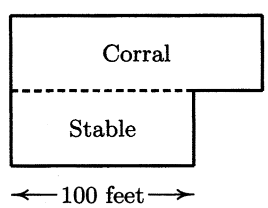


Figure 61

150. What are the value and the x-derivative of $v(w(x))$ at $x = 0$ is $w(0) = 1$, $\dfrac{dw}{dx}(0) = 5$, $v(1) = 7$, and $\dfrac{dv}{dw}(1) = -3$?

151. What are $Q(4)$ and $Q'(4)$ if $Q(x) = R(x^2)$, $R(16) = 10$ and $R'(16) = 6$?

152. Figures 62-63 show the graphs of differentiable functions $x(\text{t})$ and $y(x)$. Find the approximate value and the approximate t-derivative of $y(x(\text{t}))$ at t = 3.

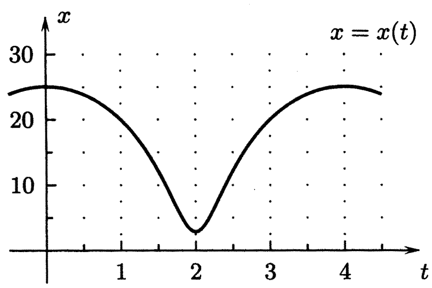


Figure 62

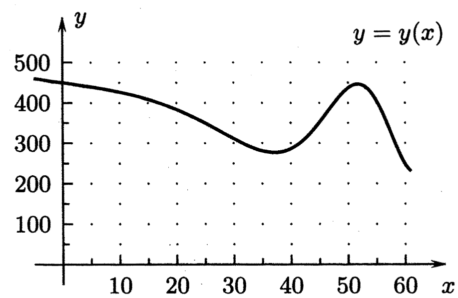


Figure 63

153. After you have driven 100 kilometers, you have 75 liters of gasoline in your car, your car is consuming gasoline at the rate of 0.2 liters per kilometer, and you are traveling 80 kilometers per hour. At what rate, measured in liters per hour, are you using gasoline at that moment?

154. At 12:00 PM a balloon is 200 meters above the ground, it has a volume of 5 liters, and it is rising 3 meters per second. Its volume increases 0.001 liter for every meter it rises. **(a)** At what rate is its volume increasing with respect to time at 12:00 PM? **(b)** Approximately how high is the balloon and what is its approximate volume 15 seconds after 12:00 PM?

155. A store sells $x(t)$ pounds of potatoes in the first t hours of a day ($0 \le t \le 12$), and it makes P(x) dollars profit on x pounds of potatoes. Express the store's profit on potatoes and the rate of change of the profit with respect to time at time t in terms of $x(t)$, $P(x)$, and their derivatives.

156. Water is flowing into a pond at the rate of 300 gallons per minute. What is the rate of change of the depth with respect to time at a time when the rate of change of the depth with respect to the volume is 0.001 feet per gallon?

157. At a temperature of 30°C, the density of mercury is 13.5217 grams per milliliter and the rate of change of the density with respect to temperature is –.0024 grams per milliliter per degree. **(a)** What is the approximate density of mercury at 30.5°C? **(b)** At what rate, with respect to time, does the density of mercury increase or decrease when it is 30°C and the temperature is falling 0.5 degrees per minute?

158. The pressure P in a balloon is a function of its volume V, the volume is a function of the temperature T, and the temperature is a function of the time. At a particular moment the rate of change of the pressure with respect to the volume is –0.01 atmospheres per liter, the rate of change of the volume with respect to the temperature is 0.2 liters per degree Celsius, and the rate of change of the temperature with respect to the time is 3 degrees Celsius per hour. What is the rate of change of the pressure with respect to time at that moment?

159. A ball rolls from left to right down the ramp with the shape of the curve $y = y(x)$ in Figure 64. **(a)** When the ball is at $x = 6$ its x-coordinate is increasing 7 feet per second. Approximately how fast is its y-coordinate increasing at that moment? **(b)** When the ball is at $x = 10$ its x-coordinate is increasing 9 feet per second. Approximately how fast is its y-coordinate decreasing at that time?

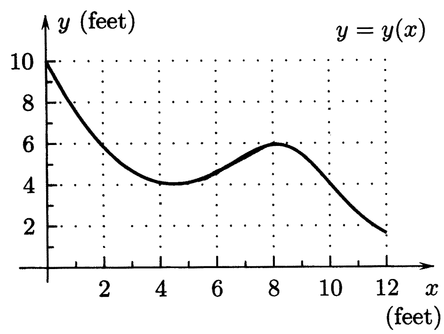


Figure 64

160. Figure 65 shows the volume $V(h)$ (thousand gallons) of the water in a lake as a function of the depth h (feet) of the water. Approximately how fast is the volume increasing, measured in gallons per hour, when there are 500,000 gallons of water in the lake if the level of the water is rising at the rate of 3 feet per day?

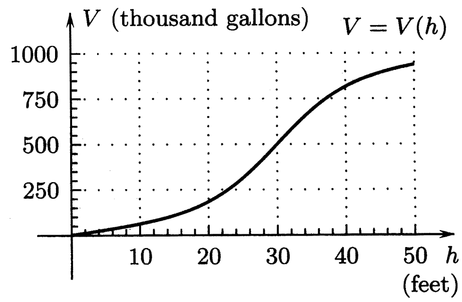


FIGURE 65

161. $\frac{dA}{dx}(6)$ where $A(x) = B(y(x))$, $y(6) = 7$, $\frac{dy}{dx}(6) = 8$, and $\frac{dB}{dx}(7) = 9$

162. $T'(10)$ where $T(t) = R((S(t))$, $S(10) = 4$, $S'(10) = 5$, and $R'(4) = -6$

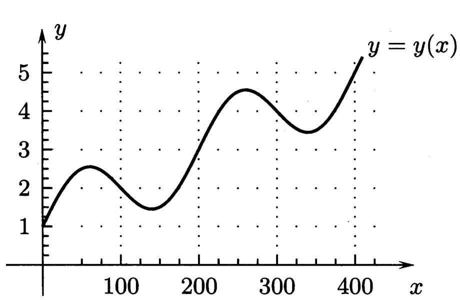
163. $\frac{dH}{dx}(2)$ where $H(x) = \mathrm{F}(x^3)$ and $\frac{dF}{dy}(8) = 10$

164. $\frac{dG}{dx}(5)$ where $G(x) = R(2x + x^2)$ and $\frac{dR}{dy}(35) = 6$

165. $p'(13)$ where $p(t) = q(r(t))$, $r(13) = 5$, $r'(13) = -7$, and $q'(5) = 10$

166. $\frac{dA}{dz}(0)$ where $A(z) = B(C(z))$, $C(0) = 3$, $C'(0) = 4$, and $B'(3) = 5$

167. What is the approximate value of $\frac{dM}{dx}(200)$ if $M(x) = K(y(x))$ and the differentiable functions $y(x)$ and $K(y)$ have the graphs in Figures 66-67?



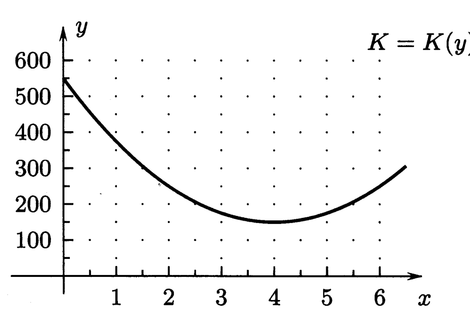


Figure 66 **Figure 67**

168. $Z'(5)$ where $Z(x) = [F(x^3)]^2$, $F(125) = 10$, $F'(5) = 6$, and $F'(125) = 8$.

169. The derivative of $xK(x^2)$ at $x = 6$ where $K(6) = 10$, $K(36) = -5$, $K'(12) = -4$, and $K(36) = 2$.

170. $\dfrac{dH}{dx}(1)$ where $H(x) = G(x^2)G(x^3)$, $G(1) = 4$, $G(2) = 10$, $G(3) = 1$, $G'(1) = -5$, $G'(2) = 13$, and $G'(3) = 9$.

171. Give approximate values of $f(1)$ and of $f'(1)$ where $f(x) = Q(Q(x))$, and the graph of $Q(x)$ is in Figure 68.

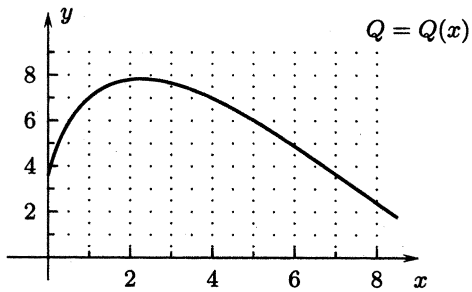


Figure 68

172. An exponential function $h(t)$ has the 1000 at $t = 0$ (years) and has the growth factor of 1.2 every five years. Give a formula for $h(t)$.

173. Find the exponential function $G(t)$ such that $G(0) = 9$ and $G(5) = 99$. (Use the growth factor from $t = 0$ to $t = 5$.)

174. What is the growth factor from A to B if A and B are positive and B is 37% greater than A?

175. A population that is given by an exponential function is 10,000 at $t = 0$ (years) and increase by 10% every three years. **(a)** What is the growth factor of the population every three years? **(b)** Give the population as a function of $t \geq 0$.

176. **(a)** By what amount does the linear function $2x + 7$ increase when x is increased by 3? **(b)** By what factor does the exponential function $7(2^x)$ increase when x is increased by 3?

177. An exponential function f(x) has the value in the next table. Give formulas for the function and its x-derivative.

x	0	1	2	3	4
$f(x)$	5	15	45	135	405

178. Give a formula for the exponential function whose graph is shown in Figure 69. Then generate the graph of the function with the graph of its derivative.

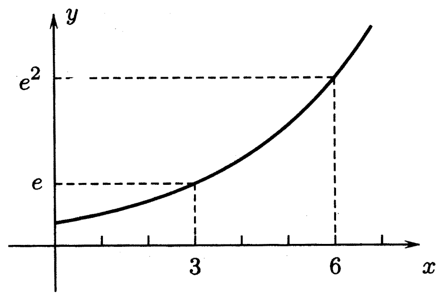


Figure 72

179. Find either a linear function or an exponential function R(t) with the values in the next table.

t	1	2	3	4	5
R(t)	95	90	85	80	75

180. Find the linear function A(t) and the exponential function B (t) such that A(0) = 20, A(1) = 10, B(0) = 20, and B(1) = 10.

181. In an exponential model of atmospheric pressure, it is assumed that the air pressure is 1035 grams per square centimeter on the surface of the earth and is halved for every 5.2 kilometers of vertical ascent. **(a)** Give a formula for air pressure p(h) (grams per square centimeter) with this model as a function of the height h (kilometers) above the earth. **(b)** What is $\frac{dp}{dh}(10.4)$?

182. If a population that grows exponentially is 500 initially and doubles every three years, at what rate is it growing after t years?

183. Uranium –238 has a half life of 4.5 x 10^9 years. **(a)** How long does it take for a five–gram sample of Uranium –238 to decay to one–fourth its original weight? **(b)** At what rate is it decaying at that time?

184. A man has 10 milligrams of lead per liter in his blood from breathing polluted air. His body eliminates the lead with a half–life of approximately 16 days. If the half-life is exactly 16 days and the man is not exposed to more lead pollution, what is the lead concentration in his blood and how rapidly is it decaying 48 days later?

185. The number of bacteria in a test tube triples every 10 hours. How many were there and at what rate were they increasing initially if 20 hours later there were 9000 bacteria in the test tube?

186. Give an equation of the tangent line to $y = 3 + 2x - e^{x}$ at its highest point.

187. Give an equation of the tangent line to $y = xe^{-x}$ at its inflection point.

188. What is the x –derivative of e^{-2z} at $x = 0$ if $Z = Z(x)$ such that $Z(0) = 3$ and $\frac{dZ}{dx}(0) = -1$?

189. What is $\frac{dR}{dx}(5)$ if $R(x) = xe^{s(x)}$, $s(5) = -4$, and $\frac{ds}{dx}(5) = 3$?

190. Which rectangle with its base on the x–axis and upper corners on $y = e^{-x^2}$ has maximum area? Give its exact area.

191. Find $\sin\theta$, $\cos\theta$, and $\tan\theta$ for the angle θ in Figure 70.

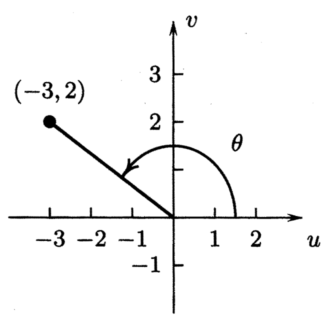


Figure 70

192. What are the coordinates (x, y) of the point Q in Figure 71 if the points P and Q are 6 units apart?

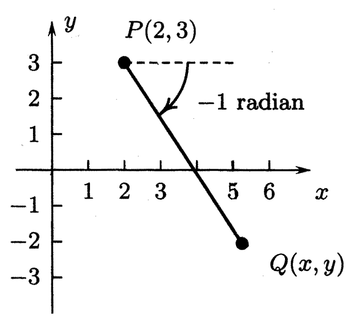


Figure 71

193. **(a)** Express h in the right triangle of Figure 72 in terms of θ. **(b)** What is $\dfrac{dh}{dt}$ at a moment when $\theta = \dfrac{1}{3}\pi$ radians and $\dfrac{d\theta}{dt} = 0.2$ radians per minute?

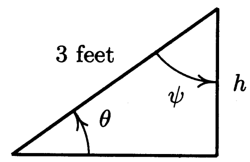


Figure 72

194. What is $\dfrac{d\psi}{dt}$ in Figure 72 at a moment when $\dfrac{d\theta}{dt} = 0.2$ radians per minute?

195. **(a) Express** ψ in Figure 72 in terms of h. (b) What is $\dfrac{d\psi}{dt}$ at a moment when $h = 2$ feet and $\dfrac{dh}{dt} = -1$ foot per minute?

196. The voltage in an electrical circuit is 220 sin ($120\pi\, t$) volts at time t (seconds). **(a)** What is the frequency of the voltage? **(b)** What is the rate of change of the voltage with respect to time at $t = 45$?

197. A right triangle has a hypotenuse of fixed length 10 feet and one of its acute angles is θ. Give the rates of change with respect to θ of its area and perimeter at $\theta = \pi/8$.

198. An acute angle x in a right triangle is changing while the leg adjacent to x has the constant length of 10 feet. Give a formula for the rate of change of the area of the triangle with respect to x.

199. The top of a 13–foot–long ladder is sliding down a tall vertical wall while its base is sliding away from the wall along the horizontal ground. When the base of the ladder is five feet from the wall, it is moving away from the wall at the rate of two feet per second. **(a)** What is the angle between the ground and the ladder at that time? **(b)** At what rate is the angle decreasing at that moment?

200. The top of a twelve–foot–long ladder is sliding down a vertical wall as its bottom is sliding on the horizontal ground. How fast are **(a)** its top and **(b)** its bottom moving when the angle between the ladder and the ground is 0.4 radians if the angle is decreasing 0.05 radians per second?

201. A five–meter–tall vertical post casts a shadow on the horizontal ground. What is the rate of change of the length of the shadow with respect to the angle x between the sun's rays and the ground when the angle is 0.8 radians?

202. An airplane is flying at an altitude of 400 feet at the speed of 200 feet per second directly away from a searchlight on the ground. At what rate is the angle between the ray of light and ground changing when the angle is 30°?

203. A man is watching a helium balloon rise vertically over his daughter's head. She released the balloon from a point 5 feet above the ground and it rises at the constant rate of 3 feet per second. Her father is 30 feet from her and his eyes are 5 feet above the ground. Give a formula for the rate of change with respect to time of the angle between his line of sight and the horizontal as a function of the time (seconds) since the release of the balloon.

204. At 10:00 AM one morning a truck driver is 50 miles east of Ozona, Texas. He drives 75 miles per hour toward the east for two hours to make a delivery outside San Antonio. Next, he drives west at 50 miles per hour for two hours to make another delivery and then drives east at 50 miles per hour for two more hours. Figure 73 is the graph of his velocity toward the east with $t = 0$ at 10 AM. **(a)** How far is he from Ozona at $t = 6$? **(b)** How is the answer to part (a) related to the areas of the rectangles in Figure 74?

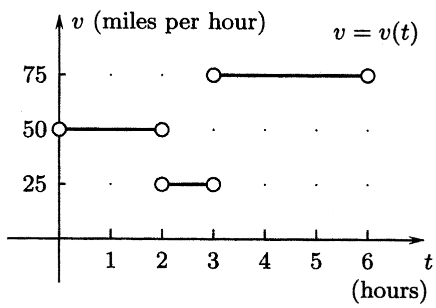


Figure 73

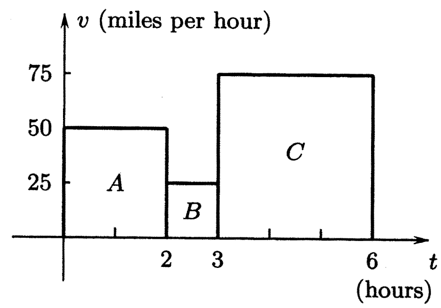


Figure 74

205. Suppose that a water tank contains 300 gallons of water at time $t = 0$ (minutes), and the rate of flow $r(t)$ (gallons per minute) into the tank for $0 \le t \le 70$ is the step function in Figure 72. **(a)** How much water flows into the tank in the time period $0 \le t \le 10$? **(b)** How much water flows out of the tank from $t = 10$ to $t = 60$? **(c)** How much water flows into the tank from $t = 60$ to $t = 70$? **(d)** How much water is in the tank at $t = 70$, and how is this number related to the areas of the four rectangles in Figure 76?

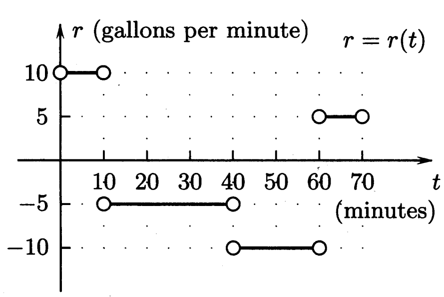


Figure 75

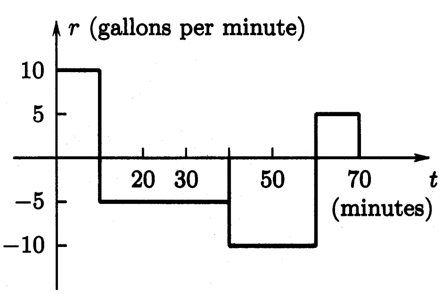


Figure 76

206. A 100–centimeter–long cylindrical core sample, drilled from the earth consists of two different types of granite at the ends and sandstone in the middle. The density *p(x)* (grams per centimeter) as a function of the distance *x* (centimeters) from one end (Figure 77) is

$$p(x) = \begin{cases} 200 & \text{for} \quad 0 < x < 30 \\ 160 & \text{for} \quad 30 < x < 80 \\ 220 & \text{for} \quad 80 < x\ 100. \end{cases}$$

(a) What is the weight of the entire core sample? **(b)** How is the weight of the core sample related to he areas of the rectangles in Figure 78?

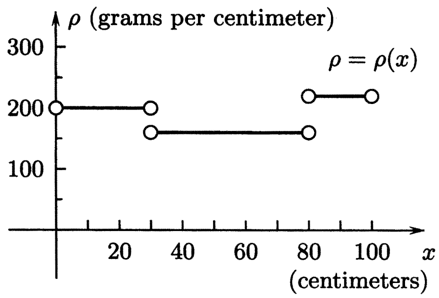


Figure 77

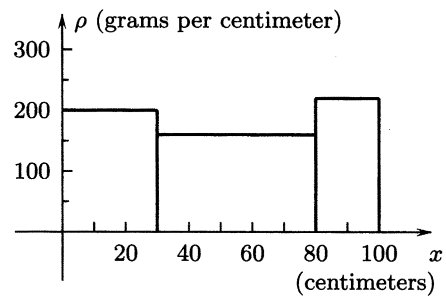


Figure 78

207. Figure 79 shows a home spa in the shape of a right circular cylinder of radius 3 feet and height 4 feet beneath a right circular cylinder of radius 5 feet and height 2 feet.

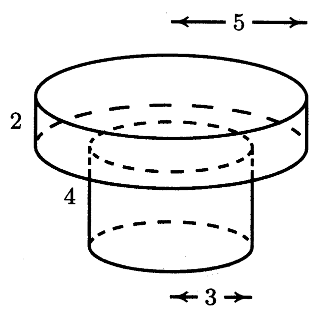
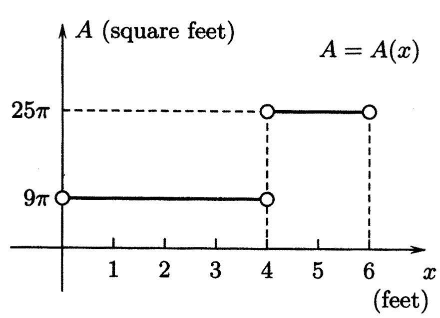


Figure 79

(a) What is the total volume of the spa? (The volume of a cylinder with base a circle of radius r and height h is $\pi r^2 h$.) **(b)** Figure 80 shows the graph of the area *A(x)* of the horizontal cross section of the jacuzzi as a function of the distance *x* of the cross section above the bottom. Because the cross section is a circle of radius 3 for $0 < x < 4$ and a circle of radius 5 for $4 < x < 6$, $A(x) = \pi(3)^2 = 9\pi$ for $0 < x < 4$ and $A(x) = \pi(5)^2 = 25\pi$ for $4 < x < 6$. How is the volume of the spa from part (a) related to the areas of the two rectangles in Figure 81?



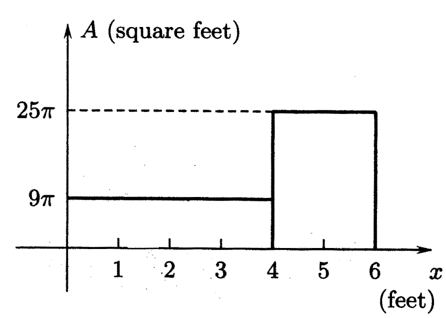


Figure 80 **Figure 81**

208. The graph of the maximum daily temperature $T(t)$ in a northern U.S. city during a week in the winter is shown in Figure 82. The time t is measured in days with $t = 0$ at 12:00 midnight Saturday night. As you can see from the graph, the maximum daily temperature was 20°F on Sunday and Monday, 10°F on Tuesday, –20°F on Wednesday and Thursday, and 40°F on Friday and Saturday.

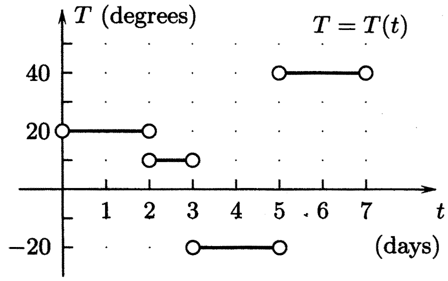


Figure 82

The maximum daily temperature was constant during each day. It was 20° F for two days, 10° F for one day, –20°F for two days, and 40°F for two days, so the average maximum temperature over the seven days was

$$\frac{1}{7}\left[20(2) + 10(1) + (-20)(2) + 40(2) = 12\frac{6}{7}\right] \text{ degrees Fahrenheit.}$$

How is the average maximum daily temperature during the week related to the areas of the four rectangles in Figure 83?

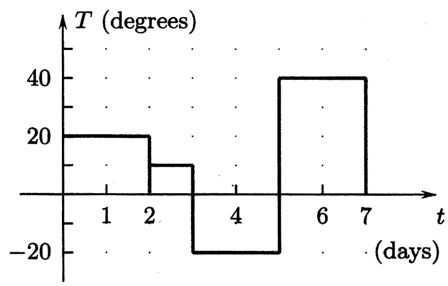


Figure 83

209. The step function $v(t)$ of Figure 81 gives velocity in the positive direction at time t of an object as it moves on an s–axis. The scale on the s–axis is given in feet, t is measured in minutes, and v is measured in feet per minute. The object is at $s = 300$ at $t = 10$. Where is it at $t = 60$?

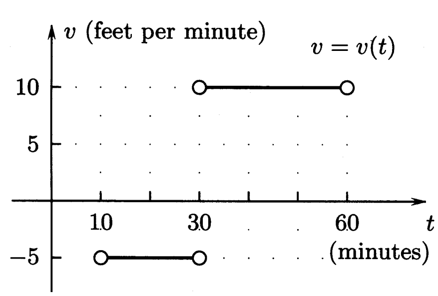


Figure 84

210. A rod extends from $x = 1$ to $x = 7$ (meters) on an x = axis, and its density $\rho(x)$ (kilograms per meter) at x is given by the step function of Figure 82. **(a)** How much does the rod weigh, measured in kilograms? **(b)** What is its average density for $1 \le x \le 7$?

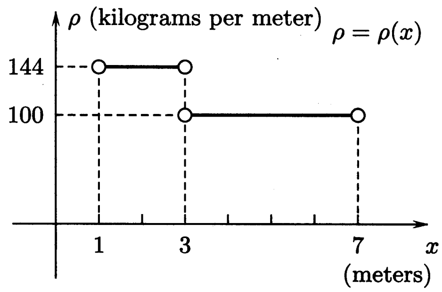


Figure 85

211. A sculpture consists of two rectangular blocks of granite, one on top of the other. The lower block is five meters high and has a square base three meters wide. The upper block is six meters high and has a square base two meters wide. The graph of the area A(x) (square meters) of the horizontal cross section of the sculpture as a function of its distance x above the ground is the step function of Figure 86. **(a)** What are the values of the numbers a and b on the A–axis in Figure 86? **(b)** What is the total volume of the sculpture?

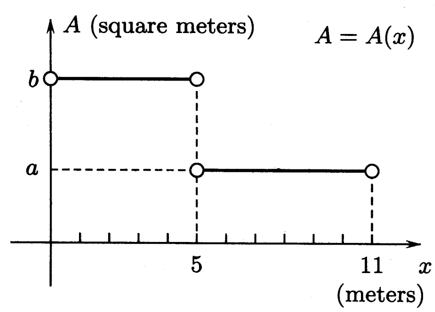


Figure 86

212. Figure 87 shows the graph of a step function $f(x)$. What is its average value for $100 \le x \le 800$?

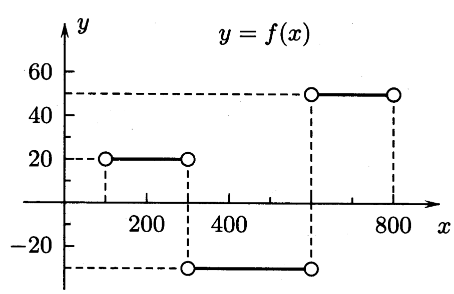


Figure 87

213. World–class jogger Ranier Schein runs eight miles between 8:30 AM and 10:00 AM. He then runs 6 miles per hour between 10:00 AM and 12:00 AM and 4 miles per hour between 12:00 PM and 1:30 PM. **(a)** How far does he run between 8:30 AM and 1:30 PM? **(b)** What is his average velocity from 8:30 AM to 1:30 PM?

214. The step function of Figure 88 is a mathematical model of the rate of rainfall $r(t)$ (inches per year) in Los Angeles from the beginning of 1881 to the beginning of 1886. **(a)** Based on this data, what was the relative change in annual rainfall in Los Angeles from the year 1883 to year 1884 and from the year 1884 to the year 1885? **(b)** What was the total rainfall in Los Angeles in the years 1881 through 1885 ? **(c)** What was the average annual rainfall in Los Angeles in the years 1881 through 1885 and how did it compare with the average annual rainfall of 14.9 inches per year from 1878 through 1992?

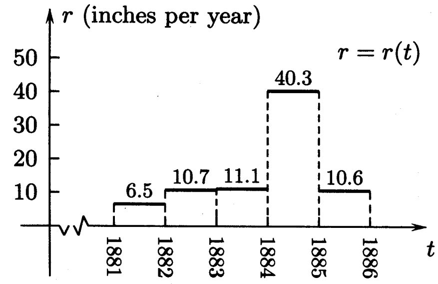


Figure 88

215. Figure 89 is a sketch of an office building, and Figure 90 shows the graph of the area $A(x)$ of the east-west, vertical cross section of the building x feet from the south end. Notice that the vertical cross section is a 30-foot-wide square for $0 < x < 60$, a 60-foot-wide square for $60 < x < 100$, and an 80-foot-wide square for $100 < x < 160$. **(a)** What is the total volume of the building. **(b)** Give the values a, b, and c of $A(x)$ in Figure 90. **(c)** The region between the graph of $A(x)$ and the x-axis for $0 < x < 160$ consists of three rectangles. How are their areas related to the volume of the building?

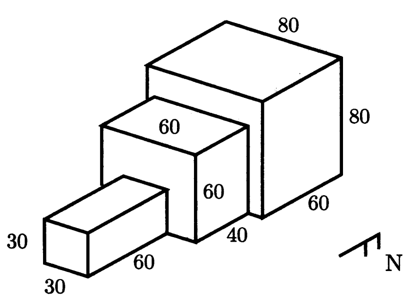


Figure 86

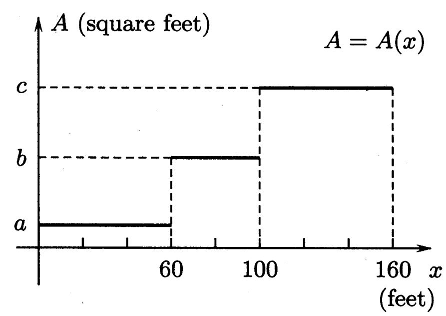


Figure 90

216. The weights at the ends of a bar bell are 10–inch long, circular cylinders of four–inch radius. The bar between them is 30 inches long and has a one–inch radius. Consequently, the cross section of the bar bell at a distance x inches from one end is a circle with a four–inch radius for $0 < x < 10$, a circle with a one–inch radius for $10 < x < 50$, and a circle with a four–inch radius for $50 < x < 60$ **(a)** Draw the graph of the area $A(x)$ of the cross section of the bar bell as a function of the distance x from one end. **(b)** What is the total volume of the bar bell and how is it related to the graph from part (a)? **(c)** How much does the bar bell weigh if it is made of steel with density 0.273 pounds per cubic inch?

217. The step function r(t) defined below gives the approximate rate of U.S. gasoline consumption, measured in millions of barrels per day, from the beginning of 1975 to the beginning of 1995. Based on this data, how much more gasoline was consumed in the U.S. in the time period $1985 \le t \le 1995$ than in the time period $1975 \le t \le 1985$? (Disregard leap years and be careful with the units.)

$$r(t)=\begin{cases} 6.2 & \text{for} \quad 1975 \le t < 1980 \\ 5.8 & \text{for} \quad 1980 \le t < 1985 \\ 6.3 & \text{for} \quad 1985 \le t < 1990 \\ 6.5 & \text{for} \quad 1990 \le t < 1995 \end{cases}$$

218. The step function $p(t)$ of Figure 91 is a mathematical model of the power produced in the entire world by wind–driven generators as a function of the year t. A kilowatt of electrical power yields one kilowatt–hour of energy in one hour. Based on the graph, how many energy (kilowatt–hours) was generated by wind–driven generators from the beginning of 1981 to the beginning of 1992, and what was the average power they generated over that time period? Be careful with the units and disregard leap years in your calculation.

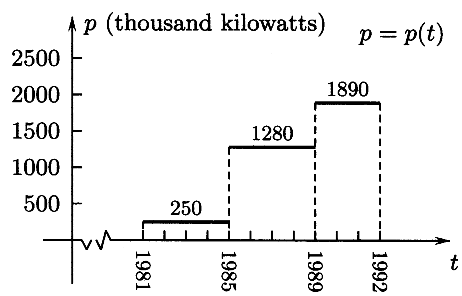


Figure 91

219. Timber is treated with creosote or other preservatives by first putting the wood in a vacuum cylinder to draw out moisture. Then the cylinder is filled with preservative and put under pressure to force the chemicals into the wood. Finally the preservative is drained from the cylinder and the vacuum reestablished to extract excess preservative from the wood. Figure 92 is the graph of the pressure during one such procedure, with the surrounding atmospheric pressure taken to be zero. What is the average pressure in the cylinder for $0 \le t \le 8$?

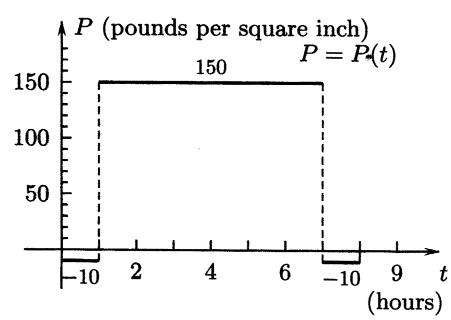


Figure 92

220. Figure 93 shows the graph of the rate of flow of water into a tank. The tank contains 100 gallons of water at $t = 0$. When for $0 \le t \le 8$ does it contain 75 gallons?

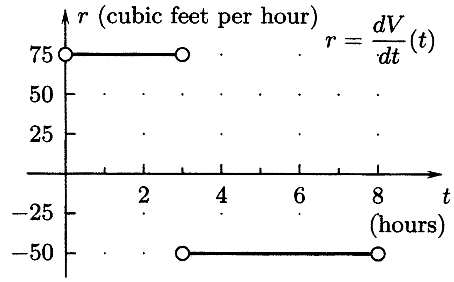


Figure 93

221. Calculate the left Riemann sum for $\int_1^8 \frac{1}{x}\, dx$ for partition $1 < 2 < 4 < 8$ of $[1, 8]$. Then draw the curve $y = 1/x$ with the rectangles whose areas equal the Riemann sum. (Notice the unequal subintervals.)

222. What is the right Riemann sum for $\int_0^{10} x^2\, dx$ for the partition of $0 < 5 < 9 < 10$ of $[0, 10]$. Draw the curve $y = x^2$ with the rectangles whose areas equal the Riemann sum. (Notice the unequal subintervals.)

223. Calculate the left and the right Riemann sums $\int_0^2 Q(x)\, dx$ for the partition of $[0, 2]$ into four equal subintervals, where Q(x) is the function whose graph is shown in Figure 94.

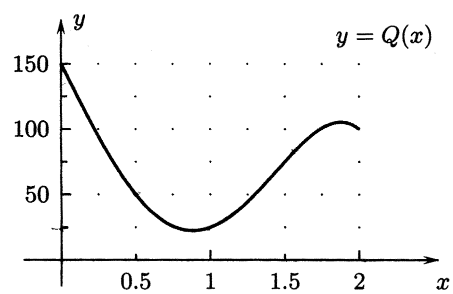


Figure 94

224. The graph of $R(x)$ is shown in Figure 95. What is the midpoint Riemann sum for $\int_0^{40} R(x)\, dx$ corresponding to the partition of $[0, 40]$ into four equal subintervals?

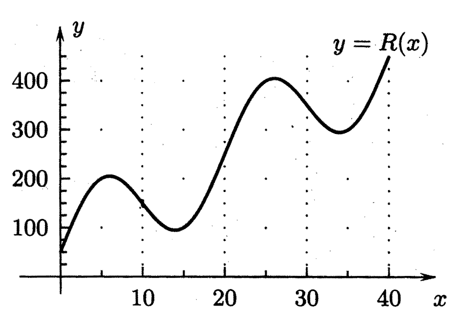


Figure 95

225. What is $f(2)$ if $f(x)$ is continuous in $[2, 6]$, $\int_2^6 \frac{df}{dx}(x)\, dx = 5$, and $f(6) = 8$?

226. Give a formula for $W(x)$ where $W'(x) = 3x^5$ and $W(1) = 2$.

227. What is $Z(4) - Z(3)$ if $Z'(t) = 4t^3$?

228. What is $W(0)$ if $W(x)$ is continuous in $[0, 300]$, $W(300) = 7$ and $\int_0^{300} W'(x)\, dx = 10$?

229. What is $G(4)$ if $G(x)$ is continuous in $[-4, 4]$, $G(-4) = 3$, and $\int_{-4}^{4} \frac{dG}{dx}(x) = 0$?

230. What is $Q(10) - Q(-10)$ if $Q(x)$ is continuous for all x and $\int_{-10}^{10} \frac{dQ}{dx}(x)\, dx = 50$?

231. A store takes in money at the rate of $10t - t^2$ dollars per hour t hours after it has opened. **(a)** How much money does it take in during its 10 hour day? **(b)** When is it taking in money at the greatest rate?

232. A ball that is rolling down a grass–covered hill has velocity $5 + \frac{3}{2}\sqrt{t}$ feet per second t seconds after it was thrown. **(a)** What is its initial velocity (its velocity at $t = 0$)? **(b)** When is its velocity 11 feet per second? **(c)** How far does it go in 16 seconds?

233. A ship is 5 nautical miles north of a lighthouse at midnight one morning and is traveling north with a velocity of $20 + \frac{1}{5}t$ knots (nautical miles per hour) at t AM. **(a)** When is its velocity 21 knots? **(b)** How far is it from the lighthouse at 10 AM?

234. Air is pumped into a balloon at the rate of $3\sqrt{t}$ cubic feet per minute from time $t = 0$ (minutes) until it breaks. Its volume is one cubic foot at $t = 0$ and it breaks when its volume is 17 cubic feet. When does it break?

235. Water flows into a tank at the rate of $10t^4 + 100t + 10$ cubic feet per day at time t (days) and leaks out at the constant rate of 6 cubic feet per day. The tank contains 25 cubic feet of water at $t = 0$. How much does it contain a day later?

236. A balloon's upward velocity is $9t - t^3$ feet per minute t minutes after it is released from six feet above the ground. **(a)** What is its upward acceleration after two minutes? **(b)** How high is it above the ground after three minutes?

237. A small airplane is flying north with the constant air speed of 125 miles per hour. The wind is blowing south at the speed of t^3 miles per hour at time t (hours) for $0 < t \le 6$. Where is the plane at $t = 6$ relative to its position at $t = 0$?

238. A ball is rolling in a straight line toward the west on a horizontal sidewalk. Its velocity toward the west is $t^3 - 4t$ feet per second at time t (seconds) for $-3 \le t \le 1$. **(a)** How far is it at $t = 1$ from its position at $t = -3$? **(b)** What is the total distance it travels from $t = -3$ to $t = 1$?

239. What is the total distance an object travels between times $t = 0$ and $t = 5$ (minutes) if its velocity is $3t - t^2$ meters per minute at time t?

240. Find the constant k such that the region between $y = \sqrt[3]{x}$ and the x–axis for $0 \le x \le k$ has area 60.75

241. An object with constant acceleration of 2 yards per minute^2 in the positive direction on an s–axis is at $s = 16$ (yards) and is traveling 4 yards per minute in the negative s–direction at $t = 0$ (minutes). When is it at $s = 12$?

242. Find the weight and center of gravity of a rod that extends from 1 to 2 on an x–axis with the scale given in meters and whose density at x is $2 + 8x^{-3}$ kilograms per meter.

243. Where is the center of gravity of a rod that extends from 0 to 1 on an x–axis with the scale given in feet and whose density at x is $x^{1/3}$ pounds per foot.

244. Use your calculator or computer to determine whether the portion of $y = x^{-2}$ for $0.1 \le x \le 1$ is shorter or longer than the portion for $1 \le x \le 100$. Give the results of your calculations.

245. In the creation of a company's logo, the two regions bounded by $y = x$ and $y = x^3$ in an xy–plane with distances measured in yards are to be painted with paint that costs five dollars per square yard. How much does the paint cost?

246. A metal plate weighing 15 kilograms per square meter has the shape of the region bounded by $y = x$ and $y = \frac{1}{8}x^4$ in an xy–plane with distances measured in meters. How much does the plate weigh?

247. An object is moving on an s–axis. Its acceleration in the positive s–direction is t feet per minute^2 at time t minutes for $0 \le t \le 10$. At $t = 0$ it is at $s = -2$ and its velocity in the positive direction is 3 feet per minute. Give a formula for its s–coordinate as function of t.

248. An object's acceleration in the positive direction on an s–axis is $3t^2$ inches per hour^2 at time t (hour) for $t \ge 0$. It is at $s = 5$ and has velocity 4 inches per hour in the positive direction at $t = 0$. Give a formula for its position as a function of t for $t \ge 0$.

249. An object's velocity in the positive direction on an s–axis is $5 - t^{-2}$ meters per minute at time t (minutes) for $t \ge 1$. It is at $s = 0$ at $t = 1$. Give formulas for its position and acceleration as functions of t for $t > 0$.

250. A ball is thrown down from a height of 32 feet above the ground with a velocity of 16 feet per second. There is no air resistance. **(a)** How long does it take to hit the ground? **(b)** What is its velocity when it hits the ground?

251. Figures 96-98 show the graphs of a car's acceleration, velocity, and position as functions of time. Find formulas for the three functions.

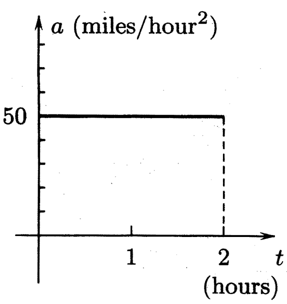


Figure 96

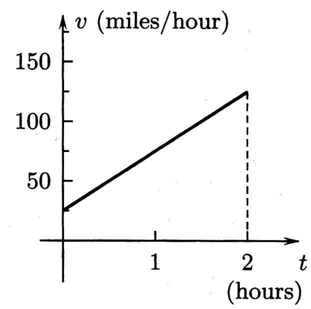


Figure 97

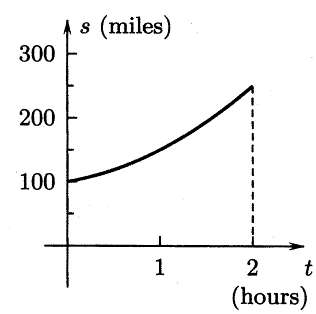


Figure 98

252. A golf ball that is dropped from a window hits the ground in three seconds. There is no air resistance. How high is the window?

253. A suitcase is dropped from a height of 100 meters. How long does it take to hit the ground if there is no air resistance?

254. A ball that is thrown into the air from four feet above the ground is 20 feet above the ground one–half second later. What was the ball's initial velocity if there was no air resistance?

255. Give a formula in terms of the positive parameter b for the area of the region bounded by $y = x^2$ and $y = b$.

256. Use an integral to find the area of the triangle with vertices $(0, 0)$, $(b, 0)$, and (b, h) for positive b and h.

257. Find the value of the parameter $b > 0$ such that the region between $y = x^2$ and $y = -x^2$ for $0 \le x \le b$ has area 18.

258. **(a)** Show that the tangent line to $y = 10x^4 - 20x^3 + 10x^2 + x$ at $x = 0$ is also its tangent line at $x = 1$. **(b)** Find the area of the region bounded by the curve and line from part (a).

259. **(a)** Give a formula in terms of the positive parameter m for the area of the region bounded by $y = \sqrt{x}$ and $y = mx$. **(b)** What is the limit of this area as $m \to 0^+$?

260. (a) Give a formula in terms of the positive parameter k for the area $A(k)$ of the region bounded by $y = x^2$ and $y = k$. **(b)** Show that $\frac{dA}{dk}(k)$ is the width of the top of the region for any choice of $k > 0$.

261. A rod that extends from $x = 0$ to $x = k$ (inches) with a positive constant k has density $x^{3/2}$ pounds per inch at x and its center of gravity is at $x = 5$. How long is the rod?

262. The intersection of a solid with an xy–plane is the region between the curves $y = x^{1/3}$ and $y = \frac{4}{3}x^{1/3}$ for $0 \le x \le 1$ in Figure 99. The cross sections of the solid perpendicular to the x–axis are circles with diameters in the xy–plane. **(a)** Describe the solid. **(b)** Find its volume. Distances are measured in feet.

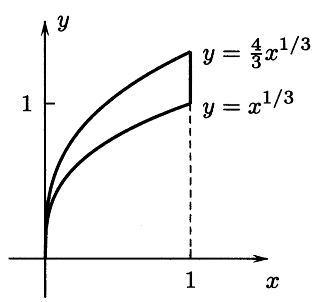


Figure 99

263. The base of a solid is the region between $y = x^3$ and the x–axis for $0 \le x \le 1$ and its cross sections perpendicular to the x–axis are squares with one side on the xy–plane. Find its volume.

264. Find the volume of a solid whose base is bounded by $y = x^2$ and $y = \sqrt{x}$ in an xy–plane and whose cross sections perpendicular to the x–axis are rectangles twice as high as they are wide.

265. Find the volume of a solid whose base is bounded by $y = x^2$ and $y = 2 - x^2$ in an xy–plane and whose cross sections perpendicular to the x–axis are semicircles with diameters in the xy–plane.

266. The base of a solid is the region between $y = 5x - 4x^2 + x^3$ and $y = 3 + 5x - 4x^2 + x^3$ in an xy–plane for $0 \le x \le 3$. Its cross sections perpendicular to the x–axis are squares with one edge on the xy–plane. What is the volume of the solid?

267. The great pyramid of Cheops at Giza in Egypt is 150 meters high and has square horizontal cross sections. The width of the cross section h meters from the top is $1.5h$ meters. What is the volume of the pyramid?

268. Figure 100 shows the graph of the area $A(h)$ of the surface of a lake as a function of the depth of the water in it. What is the approximate volume of the water in the lake when the water in it is 40 feet deep?

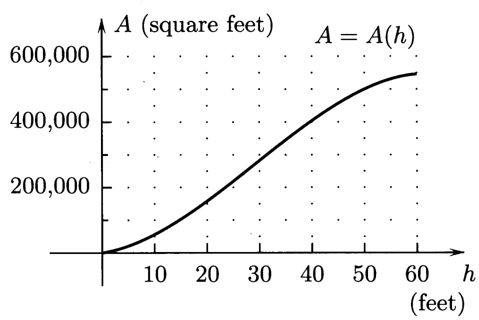


Figure 100

269. The base of a solid is the region between $y = x^3$ and $y = 2 - x^3$ for $x \ge 0$ and its cross sections perpendicular to the x–axis are squares with one edge on the xy–plane. What is the volume of the solid?

270. When the region bounded by $y = x^{5/2}$ and $y = k^2\sqrt{x}$ with a positive constant k is rotated about the x–axis, it generates a solid of volume 243π. What is the value of k?

271. The cross sections perpendicular to the x–axis of a solid are circles whose diameters extend from $y = x^3$ to $y = x^2$ for $0 \le x \le 1$ in an xy–plane. Find the volume of the solid.

272. The intersection of a solid with an xy–plane is the region between the x–axis and the parabola $y = x^2$ for $0 \le x \le 2$. The cross sections of the solid perpendicular to the x–axis are squares with one diagonal in the xy–plane. What is the volume of the solid?

273. Find the volume of the solid whose base is the semicircle between $y = \sqrt{4 - x^2}$ and the x–axis in an xy–plane and whose cross sections perpendicular to the x–axis are equilateral triangles with one side on the xy–plane.

274. The region between $y = x^2$ and $y = mx$ with $m > 0$ is rotated about the y–axis. What is the volume of the solid it generates?

275. Find the volume of the solid whose base is the region bounded by $y = 1 - x$ and $y = 1 - x^4$ in an xy–plane and whose cross sections perpendicular to the x–axis are equilateral triangles with one side on the xy–plane.

276. The region bounded by $y = x^{1/4}$, the x–axis, and $x = k$ is rotated about the x–axis. The resulting solid has volume 12π. What is the constant k?

MAC 2311 Exponential Problems Calculus I

Problems

Find the derivative of each of the following functions.

1. $V(t) = 100e^{-2t}$
2. $f(x) = 10e^{\sqrt{x}}$
3. $f(x) = e^{-2x}\cos(\pi x)$
4. $Q(x) = \dfrac{e^x}{x}$
5. $f(x) = \sqrt{e^{3x} + 1}$
6. $g(x) = x^3 + 3^x$
7. $p(t) = 40(\frac{1}{2})^t$
8. $y = Ae^{cx}$ where A,c are constants.
9. $y = e^{\sin(2x)}$
10. $y = Axe^{cx}$
11. $y = 10(e^{2x} + 3)^7$

12. The voltage drop across a component of an electric circuit is given by the function $V(t) = 10 - 10e^{-2t}$ where V(t) is measured in volts and t is time in seconds.

    A) What is the voltage drop and how fast is it changing when t = 1 second ?

    B) Graph V(t). What happens to the voltage drop in the long run?

13. In example 1 we derived a model for the population of FL as a function of time in years since 1990. The function was $P(t) = 14 * 1.03^t$.

    A) What does this model predict for the population of FL in the year 2000 and its rate of change ?

    B) What does P(11) – P(10) mean in context? How does it compare with the answer in part A.

    C) On the graph of P(t) show the values calculated in parts A and B.

14. The graph of $y = xe^{-x}$ has a horizontal tangent line at what value of x? What is special about the graph at that point?

15. Suppose that a culture of bacteria grows in such a way that it starts with 100 bacteria doubles every three hours. Write a function that gives the number of bacteria as a function of time. What does the function predict will be the number of bacteria after 30 hours and at what rate will it be growing?

More derivative practice:
Find the derivatives of the following functions.

1. $y = 2e^x + x^2$	11. $y = e^{\pi x}$
2. $y = 5^x + 2$	12. $y = ae^{bx}$
3. $y = e^{1+x}$	13. $y = \dfrac{7}{e^{4t}}$
4. $y = e^2 + x^e$	14. $y = e^{\cos(x)}$
5. $y = e^{\pi} + \pi^x$	15. $y = xe^{\cos(x)}$
6. $y = x^2 \bullet 2^x$	16. $y = e^{\sin(3x)}$
7. $y = \dfrac{2^x}{x}$	17. $y = e^x \sin(2x)$
8. $y = \dfrac{25x^2}{e^x}$	18. $y = \sec(e^{3x})$
9. $y = \dfrac{x}{2^x}$	19. $y = e^{\tan(4x)}$
10. $y = 2^{5t-3}$	20. $y = \sqrt[3]{x^2}e^{3x}$

Answers to exponential problems

1. $V'(t) = -200e^{-2t}$
2. $f'(x) = \dfrac{5e^{\sqrt{x}}}{\sqrt{x}}$
3. $f'(x) = -2e^{-2x}\cos(\pi x) - \pi e^{-2x}\sin(\pi x)$
4. $Q'(x) = \dfrac{xe^x - e^x}{x^2}$
5. $f'(x) = \dfrac{3e^{3x}}{2\sqrt{e^{3x}+1}}$
6. $g'(x) = x^3 + 3^x\ln(3)$
7. $p'(t) = 40\left(\frac{1}{2}\right)^t \ln(1/2)$
8. $y' = Ace^{cx}$
9. $y' = 2\cos(2x)e^{\sin(2x)}$
10. $y' = Ae^{cx} + Acxe^{cx}$
11. $y' = 140e^{2x}\left(e^{2x}+3\right)^6$

12.a) V(1) = 8.65 and V'(1) = 2.71
b) approaches 10.
13. A) 18.8, 0.56
B) The average rate of change in the population between 2000 and 2001. It is approximately the same.
14. x = 1. It has a turning point there.
15. $P(t) = 100(2)^{t/3}$, P(30)= 102400, P'(30) = 23659.4

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Exponential Integration

$$\int e^u du = e^u + C$$

1. $\int 4e^{3x}\sin(e^{3x})\ dx$	5. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$
2. $\int 5e^{2x}\sqrt{1+e^{2x}}\ dx$	6. $\int e^{\pi^2}\ dx$
3. $\int \frac{5}{e^{-6x}}\ dx$	7. $\int ae^{bx}\ dx$, where a and b constant
4. $\int e^{\tan(2x)}\sec^2(2x)\,dx$	8. $\int e^x(1+e^x)\ dx$

Answers:

1. $\frac{-4}{3}\cos(e^{3x})$ +C, 2. $\frac{5}{3}\sqrt{(e^{2x}+1)^3}+C$, 3. $\frac{5}{6}e^{6x}+C$, 4. $\frac{1}{2}e^{\tan(2x)}+C$, 5. $2e^{\sqrt{x}}+C$,
6. $e^{\pi^2}x+C$, 7. $\frac{a}{b}e^{bx}+C$, 8. $\frac{1}{2}\left(e^x+1\right)^2+C$